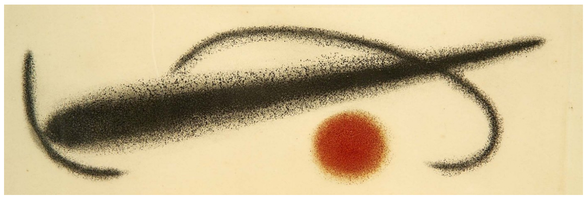
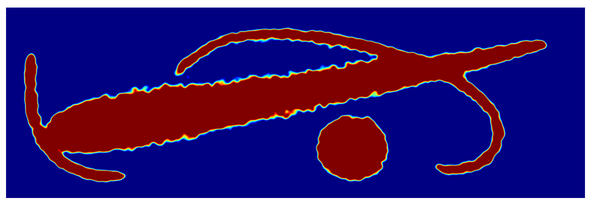
Region reconstruction from noisy samples

Emilio Vital Brazil Luiz Henrique de Figueiredo
IMPA – Instituto Nacional de Matemática Pura e Aplicada, Rio de Janeiro, Brazil


original Miró print


sample points

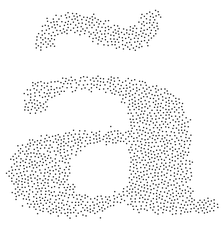

approximated region

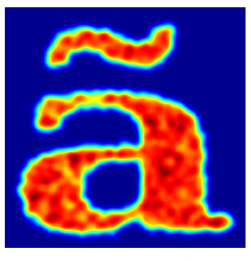

reconstructed region

Introduction

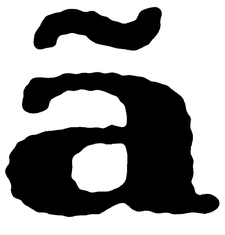
We propose a heuristic method for reconstructing a region in the plane from a noisy sample of points. The method uses radial basis functions with Gaussian kernels to compute a fuzzy membership function which provides an implicit approximation for the region.

Overview of our reconstruction method


sample points S


prereconstruction function Φ

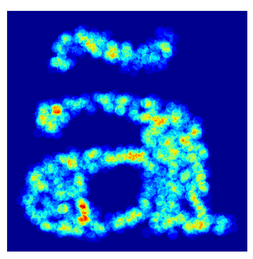

fuzzy membership function $\widehat{\chi}$

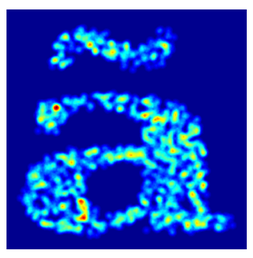

reconstructed region $\widehat{\Lambda}$

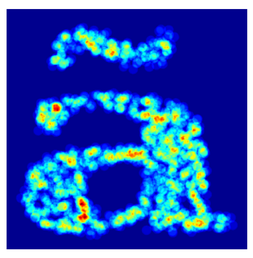
Pre-reconstruction for several kernels

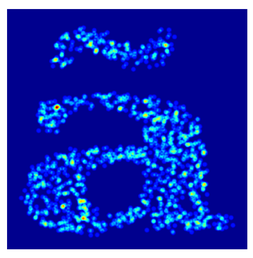

fuzzy membership function

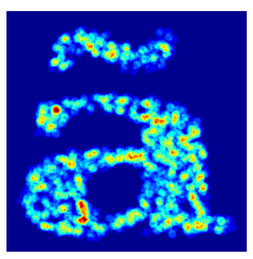

10000 sample points

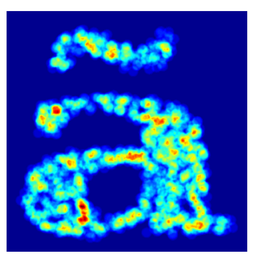

constant kernel

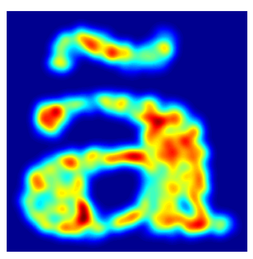

linear kernel


polynomial kernel

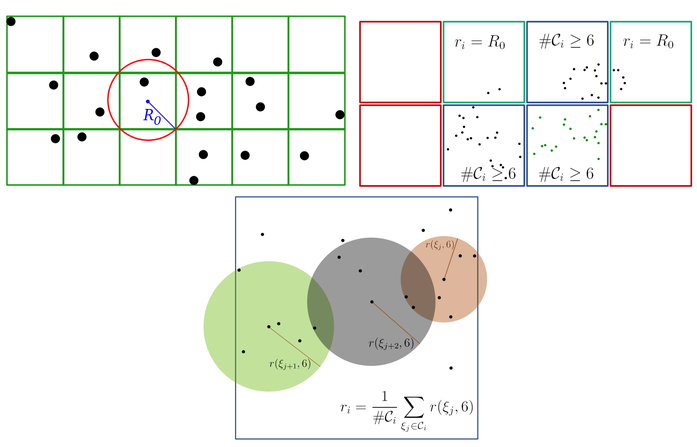

rational kernel


compact exponential kernel

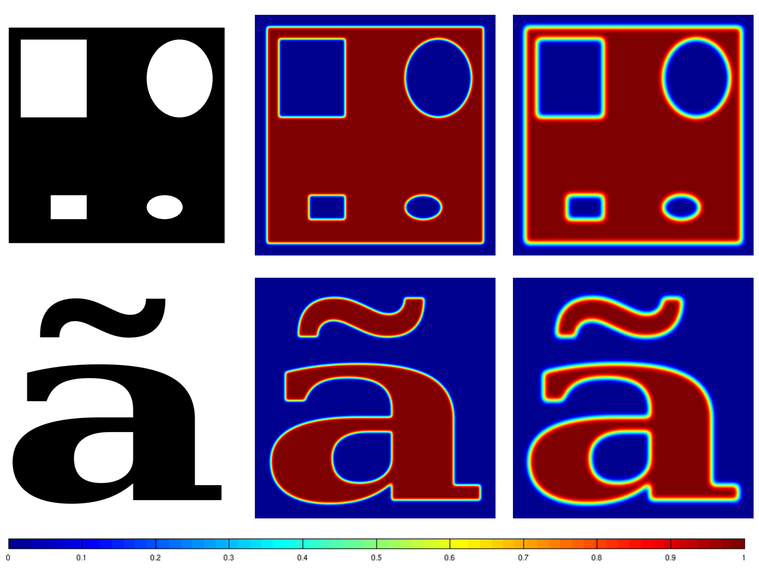

exponential kernel


Gaussian kernel

Choosing the radius

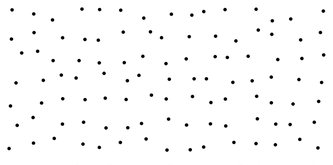


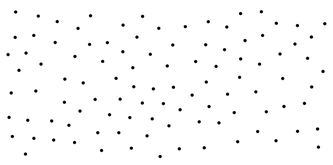
Regions used in tests



Sampling schemes


regular grid (RG)


perturbed regular grid (PRG)


Poisson disk distribution (PD)


no spatial structure (U)

Average reconstruction error

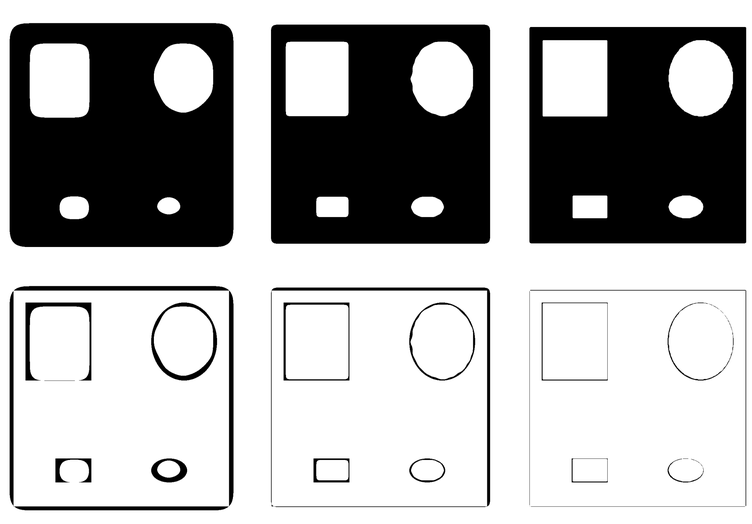
500	RG		PRG		PD		U	
χ_Q	**082**	081	088	076	092	**076**	194	172
$\widetilde{\chi}_Q$ 2%	082	081	**079**	**079**	080	079	178	172
$\widetilde{\chi}_Q$ 4%	100	099	**082**	**085**	084	089	167	171
χ_a	**061**	**058**	077	062	086	066	175	162
$\widetilde{\chi}_a$ 2%	073	072	**065**	**063**	074	069	165	162
$\widetilde{\chi}_a$ 4%	077	079	**068**	**070**	075	077	158	164

1000	RG		PRG		PD		U	
χ_Q	**050**	**047**	056	053	065	054	137	118
$\widetilde{\chi}_Q$ 2%	060	056	**056**	**056**	059	059	123	119
$\widetilde{\chi}_Q$ 4%	091	082	**073**	**068**	073	070	120	123
χ_a	**041**	**040**	052	043	055	043	116	102
$\widetilde{\chi}_a$ 2%	055	054	**047**	**047**	048	047	106	104
$\widetilde{\chi}_a$ 4%	071	067	**058**	**055**	058	056	102	107

4000	RG		PRG		PD		U	
χ_Q	**027**	**027**	036	034	040	038	060	047
$\widetilde{\chi}_Q$ 2%	047	040	**044**	**040**	045	042	052	050
$\widetilde{\chi}_Q$ 4%	088	078	071	056	070	057	**065**	**056**
χ_a	**028**	**027**	032	030	032	029	050	041
$\widetilde{\chi}_a$ 2%	049	046	**038**	**035**	038	036	043	042
$\widetilde{\chi}_a$ 4%	072	063	061	049	057	**046**	**052**	048

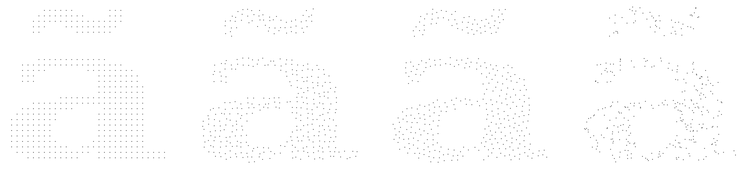
10000	RG		PRG		PD		U	
χ_Q	**022**	**021**	025	023	028	027	037	030
$\widetilde{\chi}_Q$ 2%	051	044	042	036	042	036	**036**	**033**
$\widetilde{\chi}_Q$ 4%	084	072	072	055	070	053	**057**	**040**
χ_a	**018**	**018**	021	021	023	022	031	026
$\widetilde{\chi}_a$ 2%	046	043	036	031	035	030	**031**	**024**
$\widetilde{\chi}_a$ 4%	069	058	061	047	058	044	**047**	**035**

Smoothing effect on boundaries

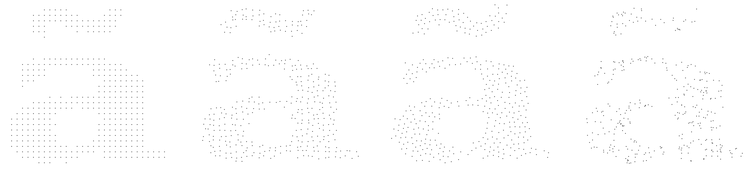


Samples used in tests

1000 points, noiseless



1000 points, noisy



10000 points, noiseless

10000 points, noisy

RG PRG PD U

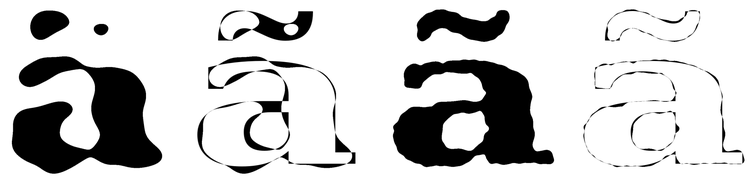
Reconstructed regions and reconstruction errors for samples with 2% noise

regular grid (RG)

perturbed regular grid (PRG)

Poisson disk distribution (PD)

no spatial structure (U)



1000 points 10000 points