



SIBGRAPI

2022

35th Conference on Graphics,
Patterns and Images

Region reconstruction with the sphere-of-influence diagram

Luiz Henrique de Figueiredo



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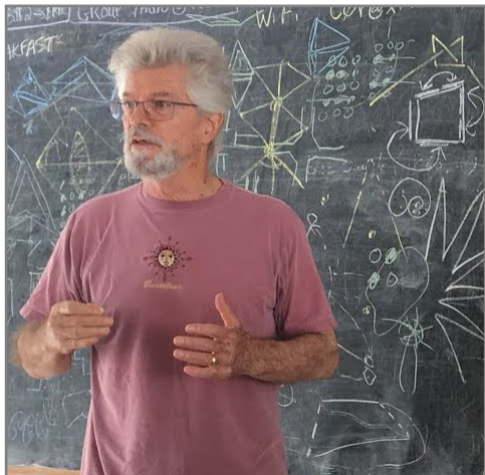
Region reconstruction with the sphere-of-influence diagram

Luiz Henrique de Figueiredo

with Afonso Paiva



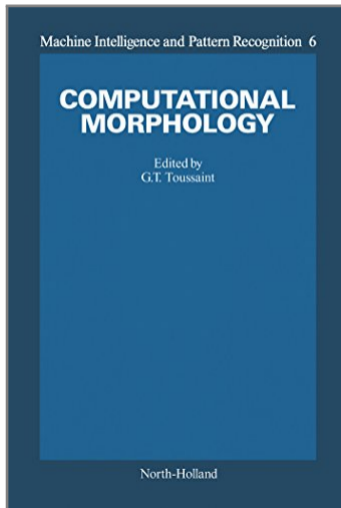
Godfried Toussaint (1944–2019)



at Bellairs Workshop on Computational Geometry, Barbados, 2019
photo by Yushi Uno

Pioneer in **Computational Geometry**,
the study of algorithms for geometric problems

Godfried Toussaint (1944–2019)



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the study of algorithms for geometric problems

Coined the term **Computational Morphology**

Godfried Toussaint (1944–2019)

Computational Morphology

A Computational Geometric Approach
to the Analysis of Form

Edited by

Godfried T. TOUSSAINT
*School of Computer Science
McGill University
Montreal, Quebec
Canada*

1988

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“When a computational geometric structure or operator is intended to extract the shape or form of an object to which it is applied, it is referred to as a problem in computational morphology.”

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Given a set of points S in the plane,
what is the shape of S ?

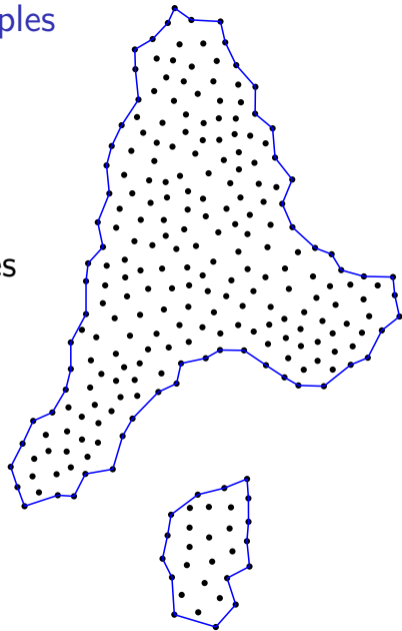
Region reconstruction from point samples

point sample



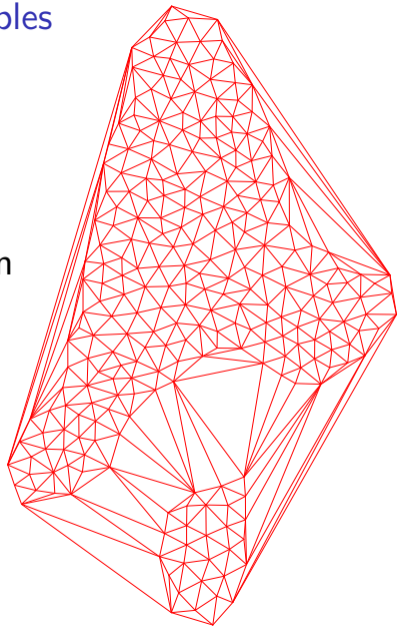
Region reconstruction from point samples

region boundaries



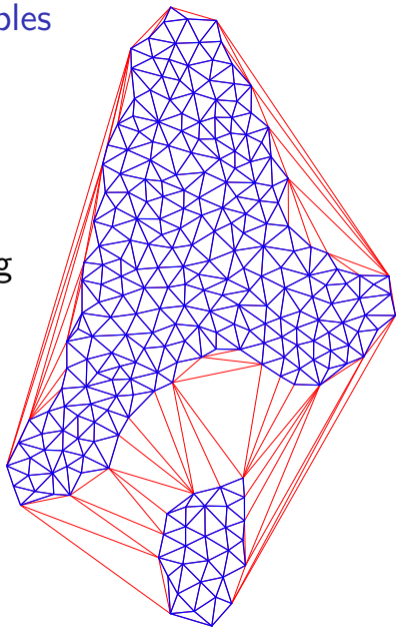
Region reconstruction from point samples

Delaunay triangulation



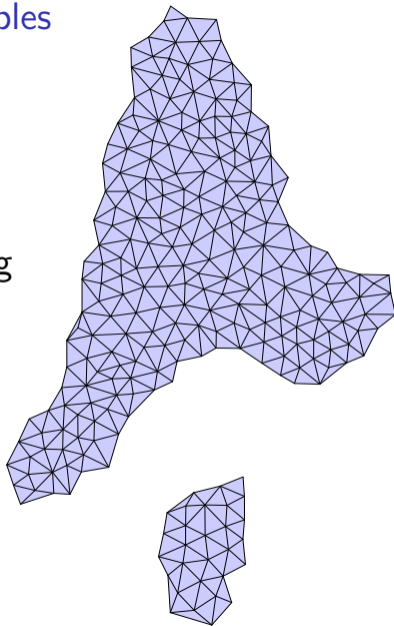
Region reconstruction from point samples

Delaunay filtering



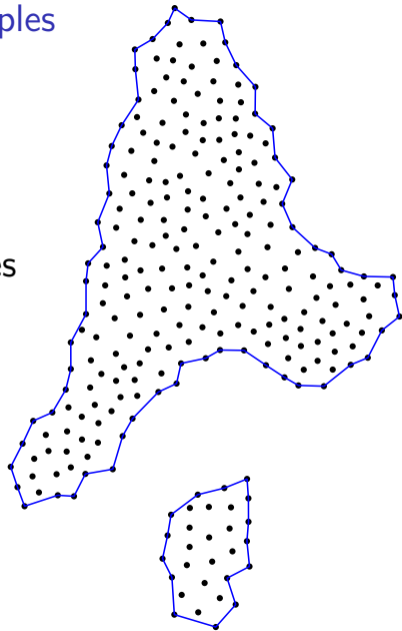
Region reconstruction from point samples

Delaunay filtering



Region reconstruction from point samples

region boundaries



Spheres of influence

Toussaint (1988)

point sample



Spheres of influence

Toussaint (1988)

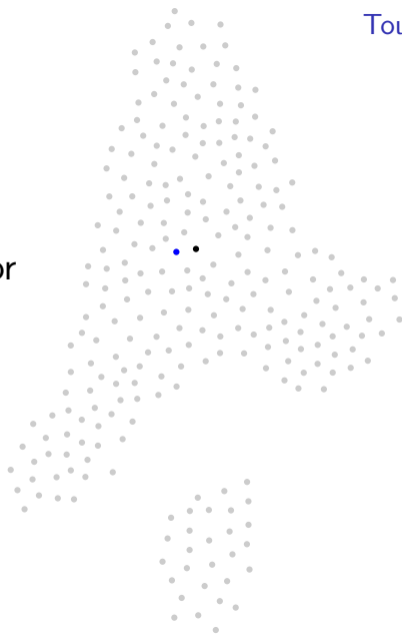
a point



Spheres of influence

Toussaint (1988)

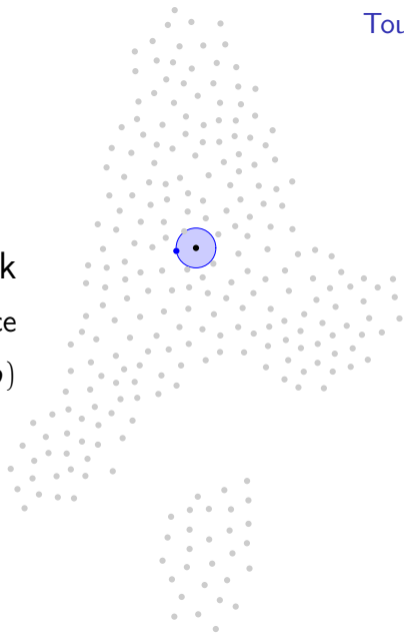
nearest neighbor



Spheres of influence

Toussaint (1988)

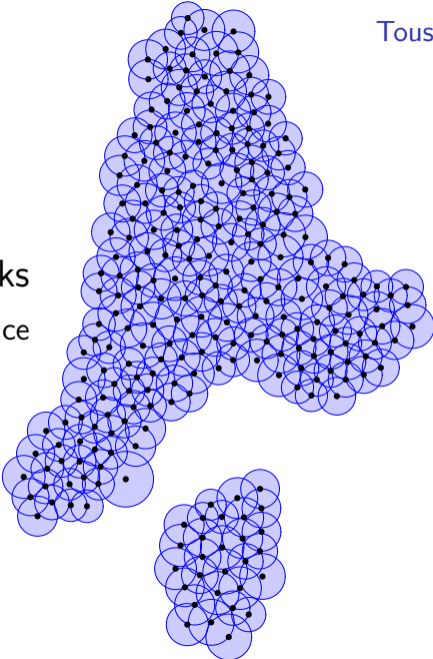
nearest neighbor disk
sphere of influence
radius $r(p)$



Spheres of influence

Toussaint (1988)

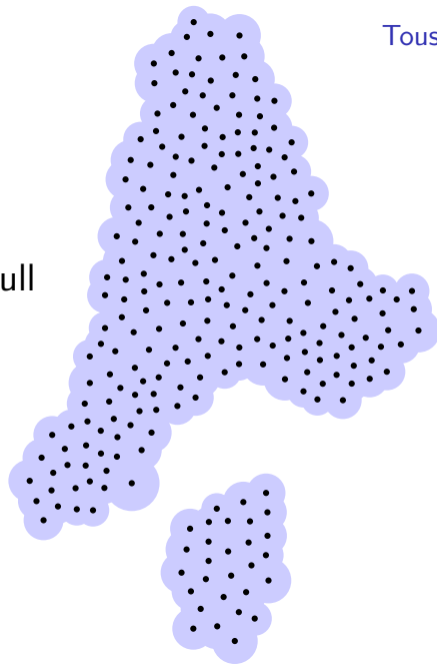
nearest neighbor disks
spheres of influence



Spheres of influence

Toussaint (1988)

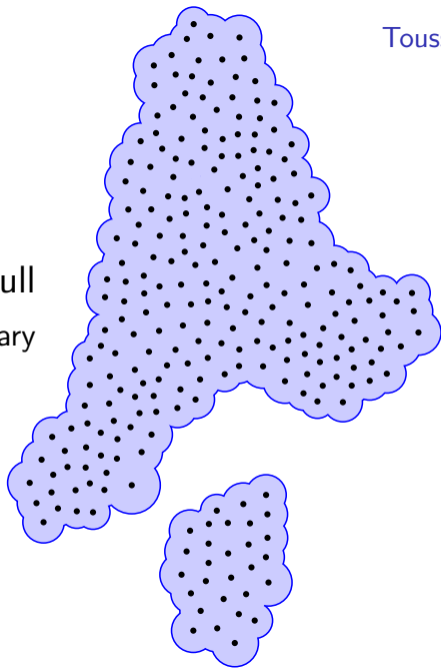
sphere-of-influence hull



Spheres of influence

Toussaint (1988)

sphere-of-influence hull
hard to find boundary



Spheres of influence

Toussaint (1988)

sphere-of-influence graph
mutual influence

$$\text{dist}(p, q) \leq r(p) + r(q)$$



Spheres of influence

Toussaint (1988)

sphere-of-influence graph

mutual influence

not planar

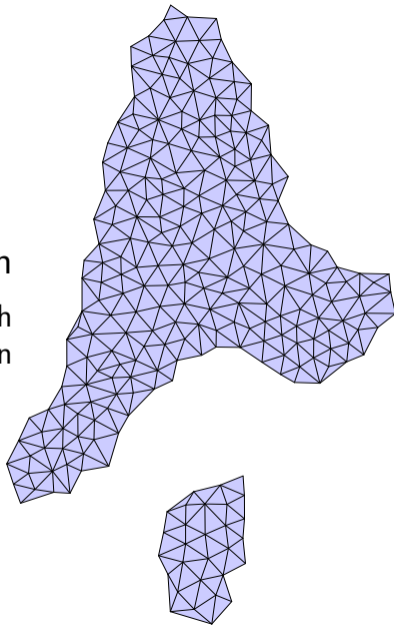
$$\text{dist}(p, q) \leq r(p) + r(q)$$



Spheres of influence

sphere-of-influence diagram

= sphere-of-influence graph
 \cap Delaunay triangulation

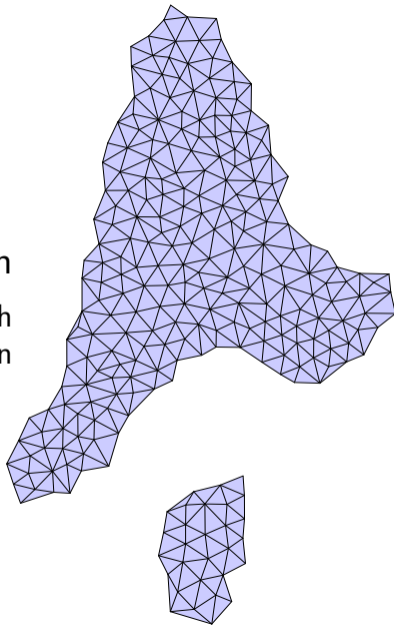


Spheres of influence

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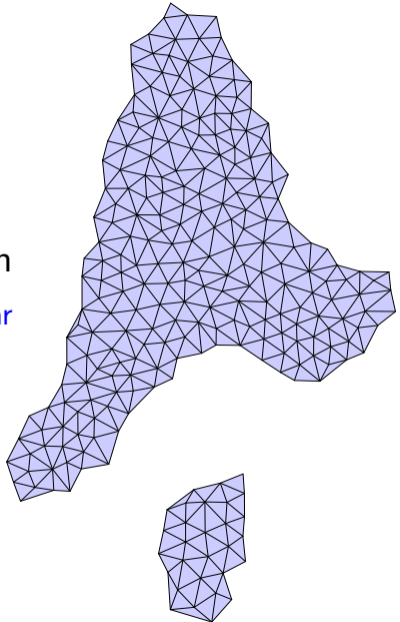
Delaunay filtering



Spheres of influence

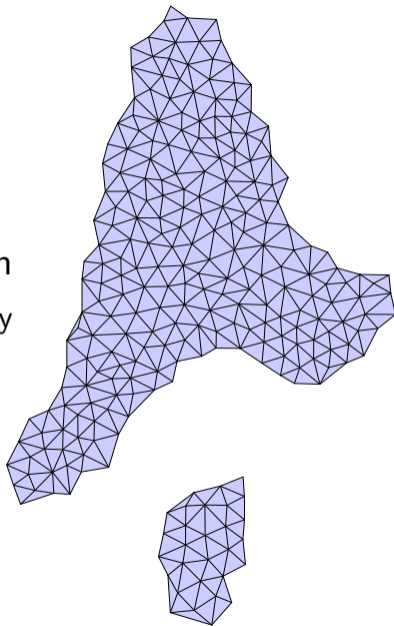
sphere-of-influence diagram

planar



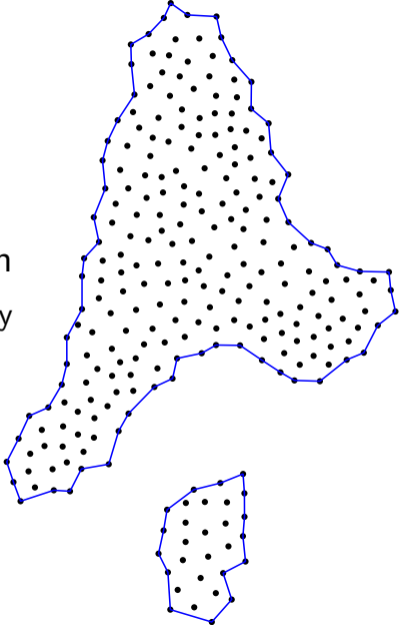
Spheres of influence

sphere-of-influence diagram
easy to find boundary



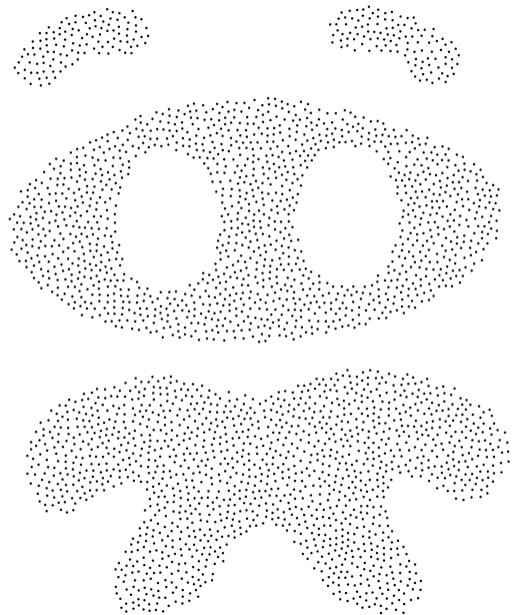
Spheres of influence

sphere-of-influence diagram
easy to find boundary



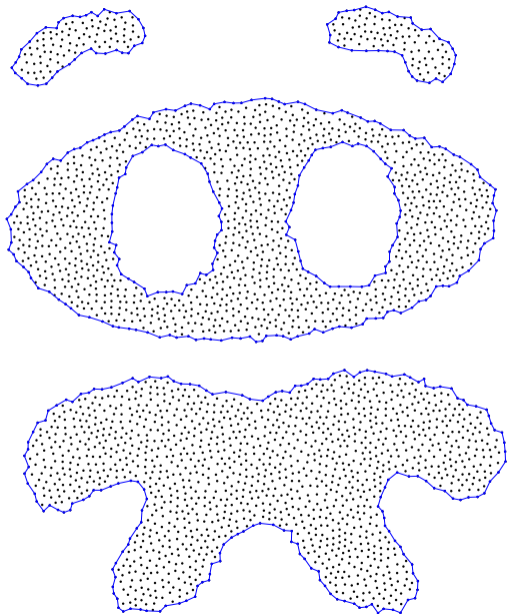
Reconstructing well-distributed samples

point sample

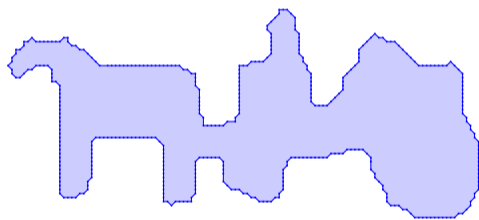
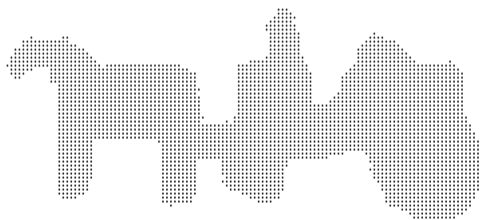


Reconstructing well-distributed samples

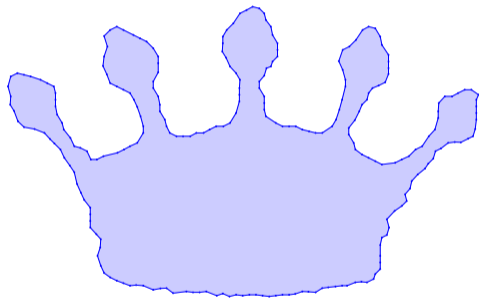
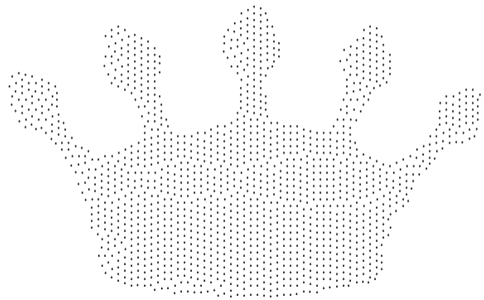
reconstructed boundaries



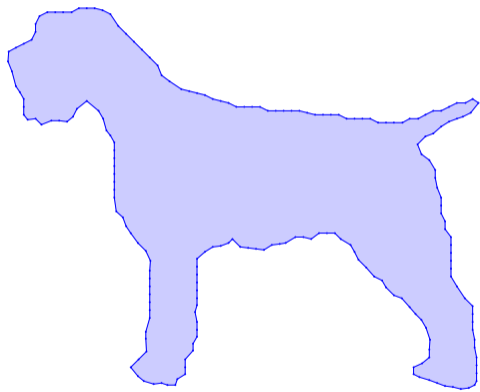
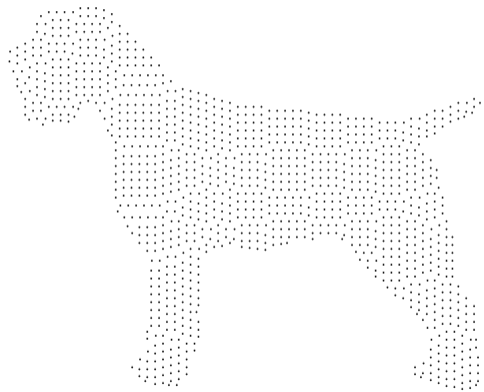
Reconstructing well-distributed samples



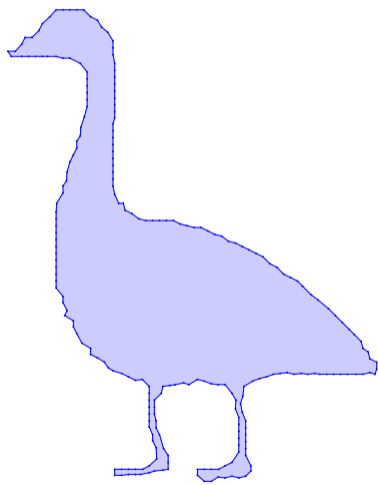
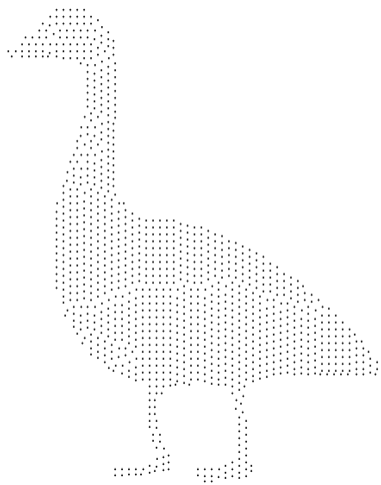
Reconstructing well-distributed samples



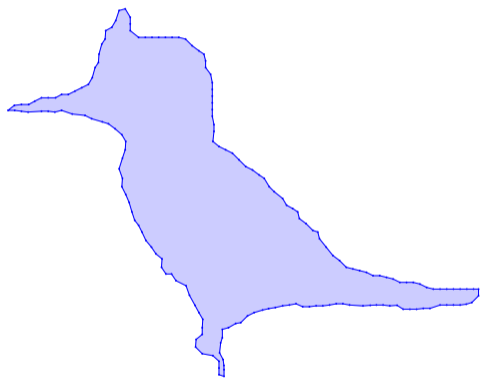
Reconstructing well-distributed samples



Reconstructing well-distributed samples



Reconstructing well-distributed samples



Effect of sampling density

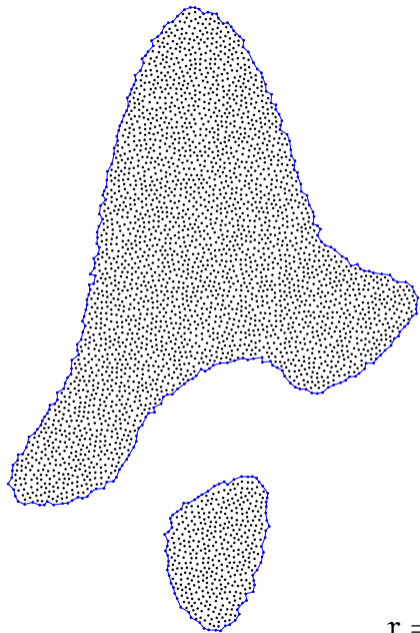


Effect of sampling density



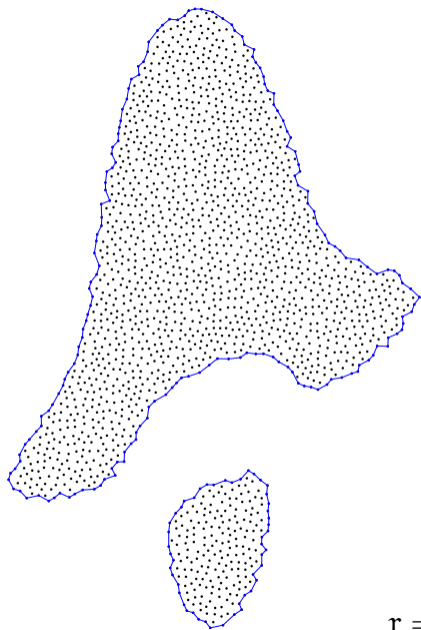
$r = 0.005$ $n = 7270$

Effect of sampling density



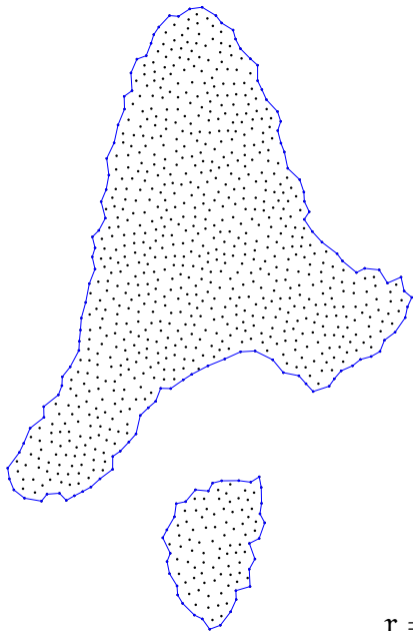
$r = 0.007$ $n = 3685$

Effect of sampling density



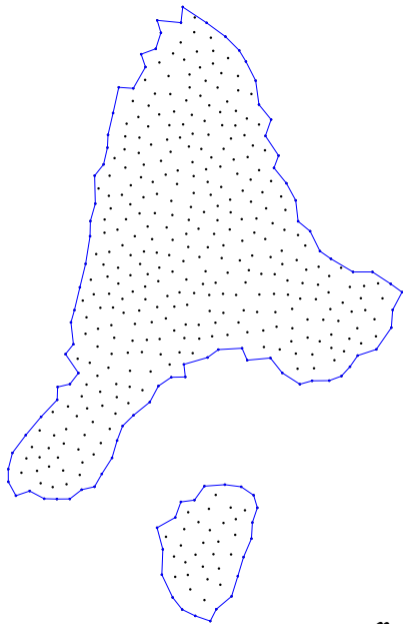
$r = 0.010$ $n = 1814$

Effect of sampling density



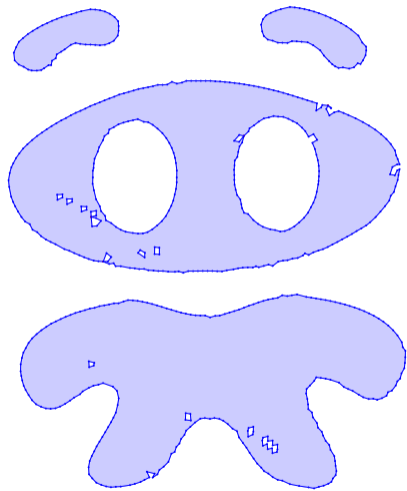
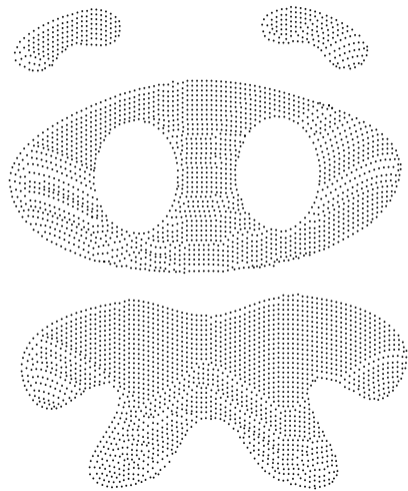
$r = 0.014$ $n = 944$

Effect of sampling density

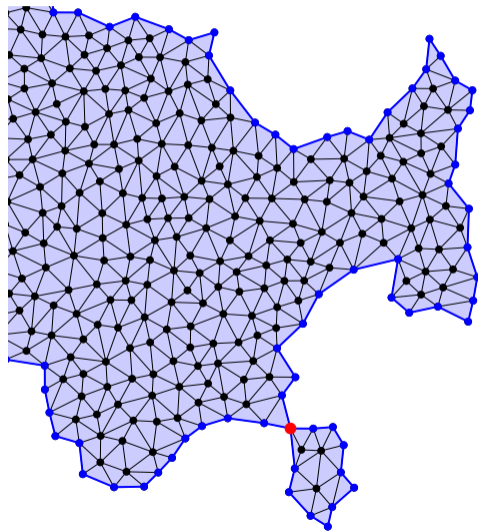
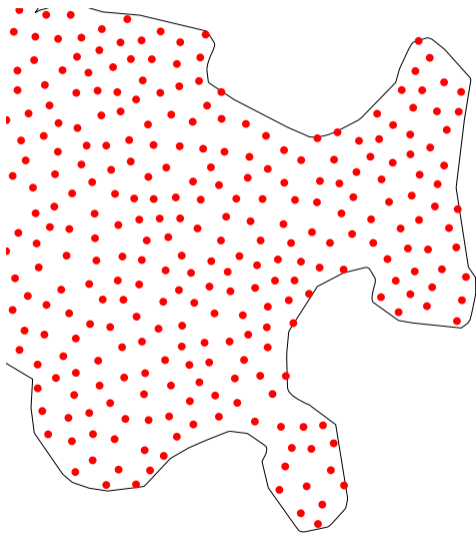


$r = 0.020$ $n = 451$

Effect of sample quality – false holes



Effect of sample quality – topological irregularities



Tuning parameter μ

scaled nearest neighbor radius of a sample point p

Radke (1988)

$$R(p) = \mu r(p)$$

Tuning parameter μ

scaled nearest neighbor radius of a sample point p

Radke (1988)

$$R(p) = \mu r(p)$$

uv edge of sphere-of-influence diagram when scaled disks intersect

$$\text{dist}(u, v) \leq R(u) + R(v)$$

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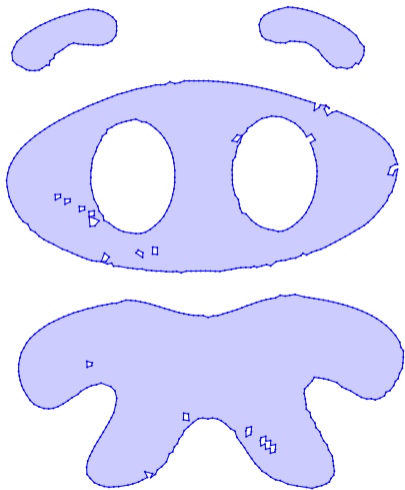
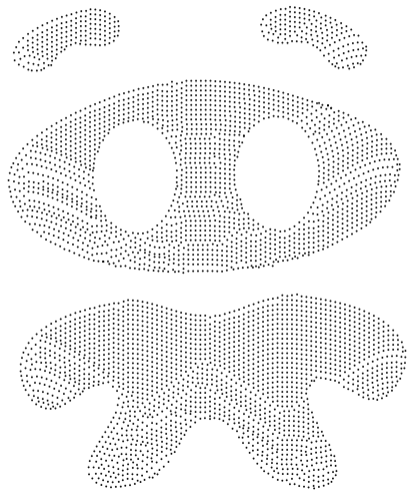
uvw face of sphere-of-influence diagram when

threshold

$$\mu \geq \mu(u, v, w) = \max(\mu(u, v), \mu(v, w), \mu(w, u))$$

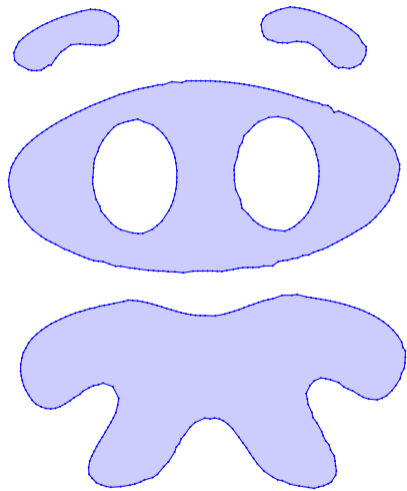
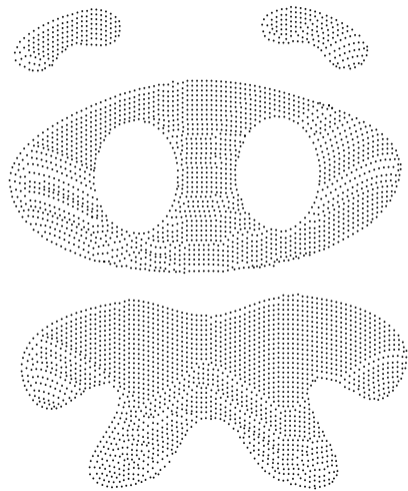
Effect of sample quality

$\mu = 1$



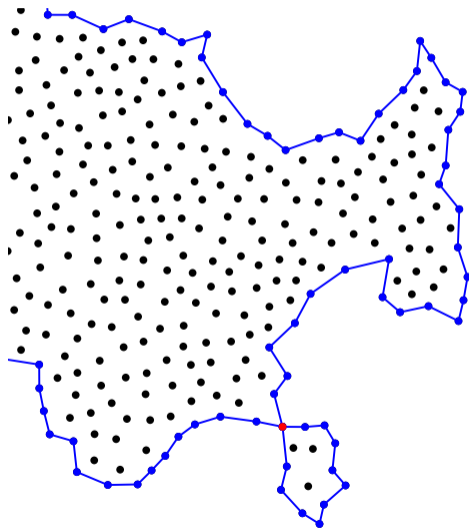
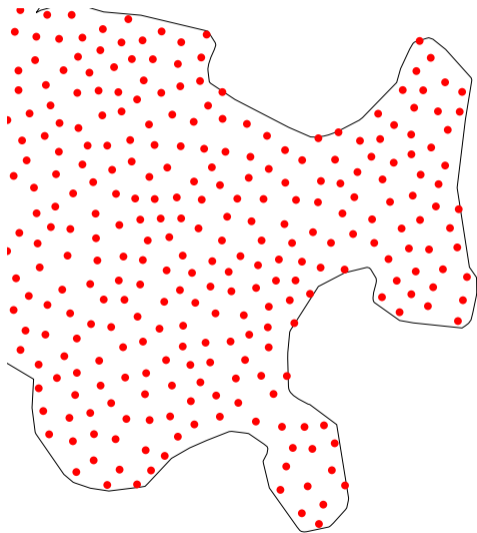
Effect of sample quality

$\mu = 1.26$



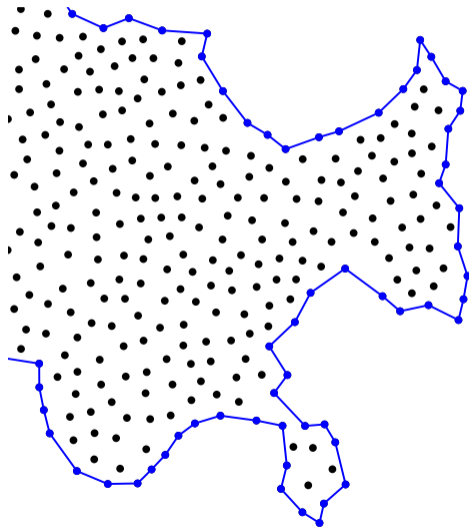
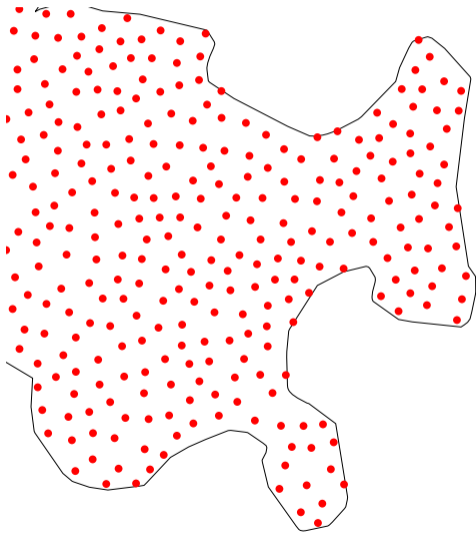
Effect of sample quality

$\mu = 1$



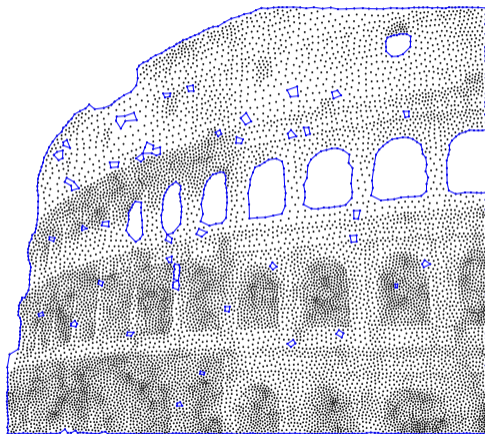
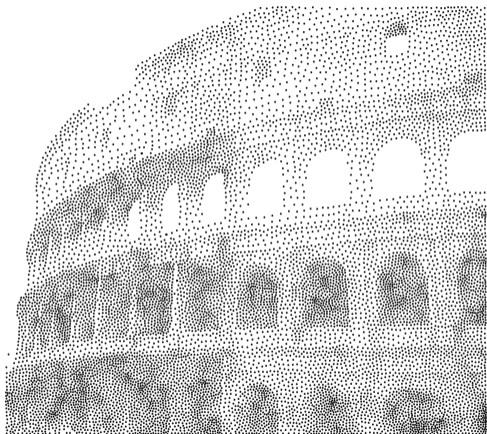
Effect of sample quality

$\mu = 1.09$



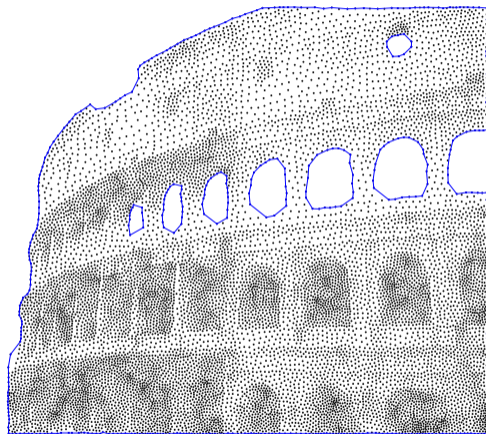
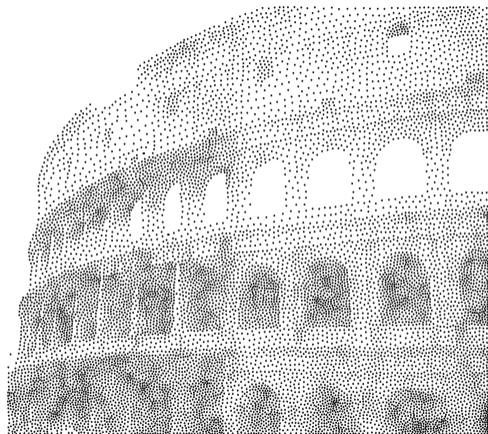
Effect of sample quality

$\mu = 1$



Effect of sample quality

$\mu = 1.42$



Related work

- α -shapes

Edelsbrunner, Kirkpatrick, Seidel (1983)

- χ -shapes

Duckham, Kulik, Worboys, Galton (2008)

- CT-shapes

Thayyil, Parakkat, Muthuganapathy (2020)

Related work

- α -shapes

Edelsbrunner, Kirkpatrick, Seidel (1983)

qualitative comparison

- χ -shapes

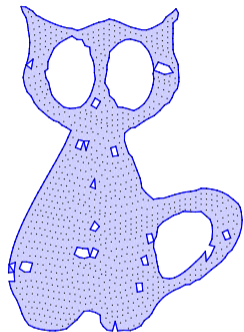
Duckham, Kulik, Worboys, Galton (2008)

- CT-shapes

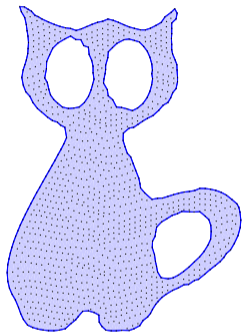
Thayyil, Parakkat, Muthuganapathy (2020)

quantitative comparison

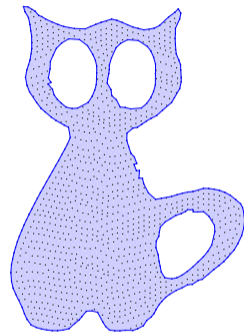
Comparison with α -shapes



critical $\alpha = 19.69$



$\alpha = 21.30$



$\mu = 1.20$

Comparison with α -shapes

α -threshold of Delaunay triangle with sides a, b, c is radius of circumcircle

$$\alpha \geq \frac{abc}{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}$$

Comparison with α -shapes

α -threshold of Delaunay triangle with sides a, b, c is radius of circumcircle

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Comparison with α -shapes

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$$\alpha \geq \frac{abc}{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}$$

not scale invariant

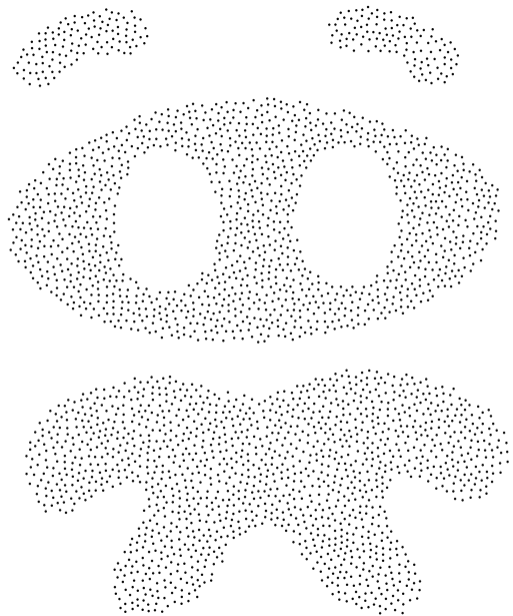
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scale invariant

α -spectrum versus μ -spectrum

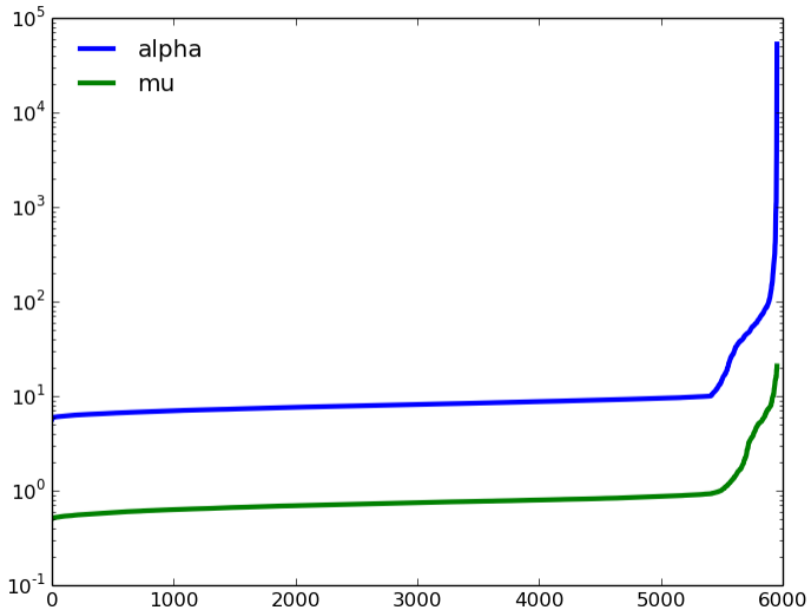


α -spectrum

min 5.73

max 52923.99

not scale invariant

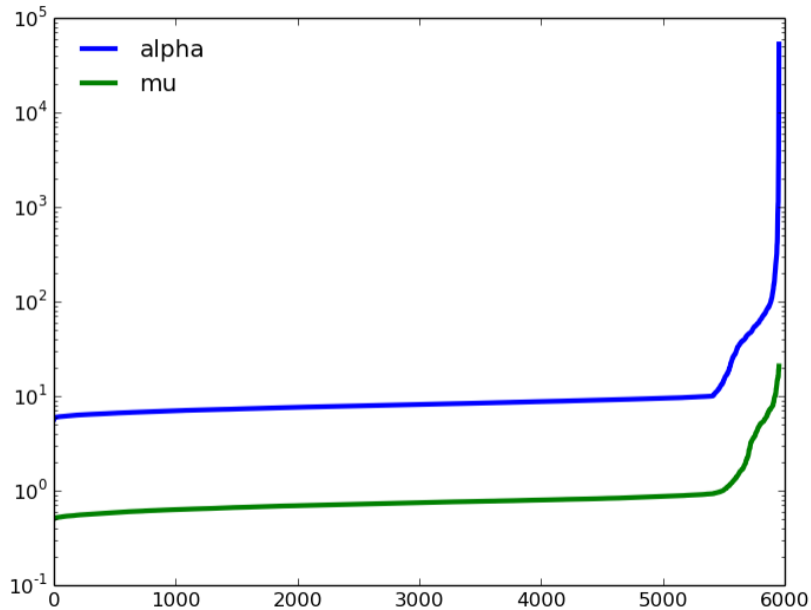


μ -spectrum

min 0.50

max 20.80

scale invariant



Conclusion

The sphere-of-influence diagram. . .

- **reconstructs** regions from well-distributed point samples, already for $\mu = 1$
- automatically identifies **connected components and holes**
- supports a **scaling parameter** μ to handle samples of lower spatial quality, choosing a suitable μ is easier than choosing a suitable α
- is a **natural subset** of the Delaunay triangulation, **extracted in linear time**
- relies on a **simple geometric predicate** for Delaunay filtering



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Luiz Henrique de Figueiredo

with Afonso Paiva

