SIBGRAPI'18

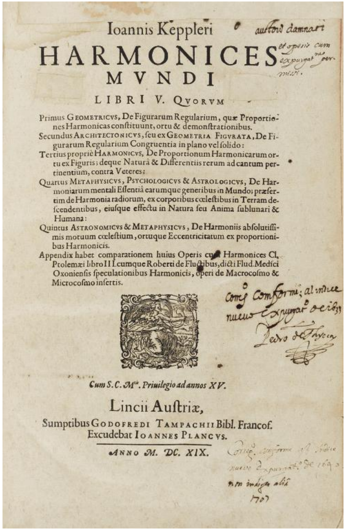
31st Conference on Graphics, Patterns and Images

Synthesizing Periodic Tilings of Regular Polygons

Asla Medeiros e Sá · FGV
Luiz Henrique de Figueiredo · IMPA
José Ezequiel Soto Sánchez · IMPA

Motivation: tile the plane with regular polygons

Kepler (1619)



Ioannis Keppleri

HARMONICES MVNDI

LIBRI V. QVORVM

Primus GEOMETRICVS, De Figurarum Regularium, quæ Proportiones Harmonicas constituunt, ortu & demonstrationibus.

Secundus ARCHITECTONICVS, seu ex GEOMETRIA FIGVRATA, De Figurarum Regularium Congruentia in plano vel solido:

Tertius propriè HARMONICVS, De Proportionum Harmonicarum ortu ex Figuris; deque Naturâ & Differentiis rerum ad cantum pertinentium, contra Veteres:

Quartus METAPHYSICVS, PSYCHOLOGICVS & ASTROLOGICVS, De Harmoniarum mentali Essentiâ earumque generibus in Mundo; præsertim de Harmonia radiorum, ex corporibus cœlestibus in Terram descendentibus, eiusque effectu in Natura seu Anima sublunari & Humana:

Quintus ASTRONOMICVS & METAPHYSICVS, De Harmoniis absolutissimis motuum cœlestium, ortuque Eccentricitatum ex proportionibus Harmonicis.

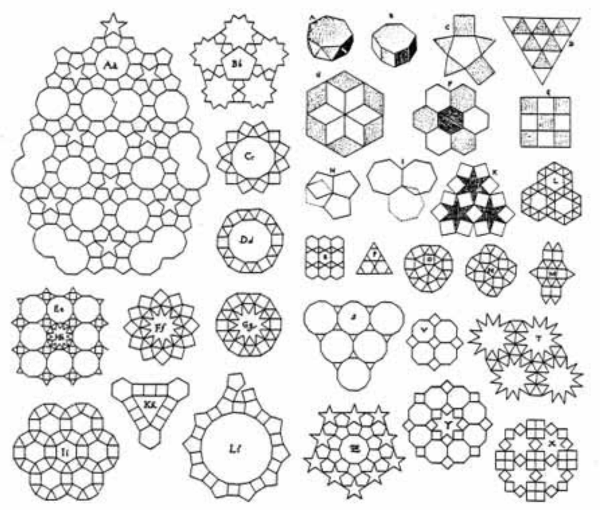
Appendix habet comparationem huius Operis cum Harmonices Cl. Ptolemæi libro III. cumque Roberti de Fluctibus, dicti Flud. Medici Oxoniensis speculationibus Harmonicis, operi de Macrocosmo & Microcosmo insertis.

Cum S. C. M^tis. Priuilegio ad annos XV.

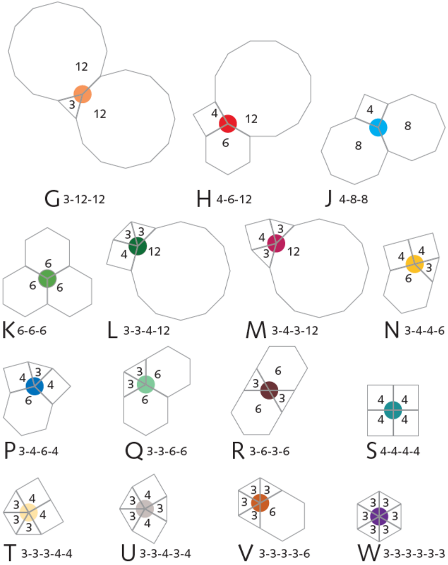
Lincii Austriæ,

Sumptibus GODOFREDI TAMPACHII Bibl. Francof. Excudebat IOANNES PLANCVS.

ANNO M. DC. XIX.



Rigidity: only 15 vertex neighborhoods



Rigidity: only 11 tilings are 1-regular

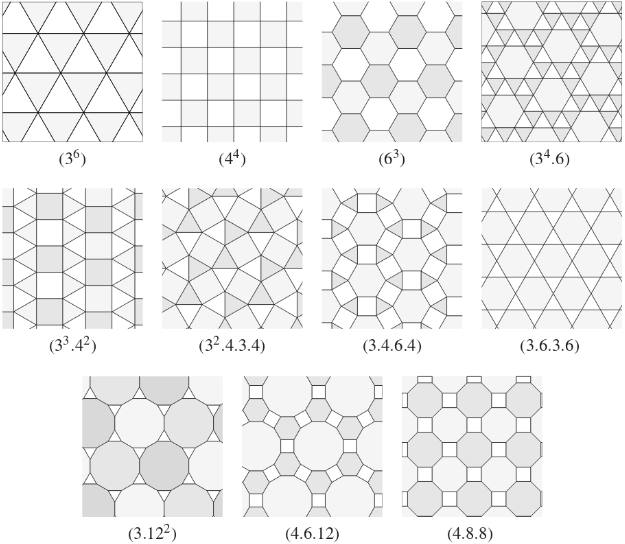
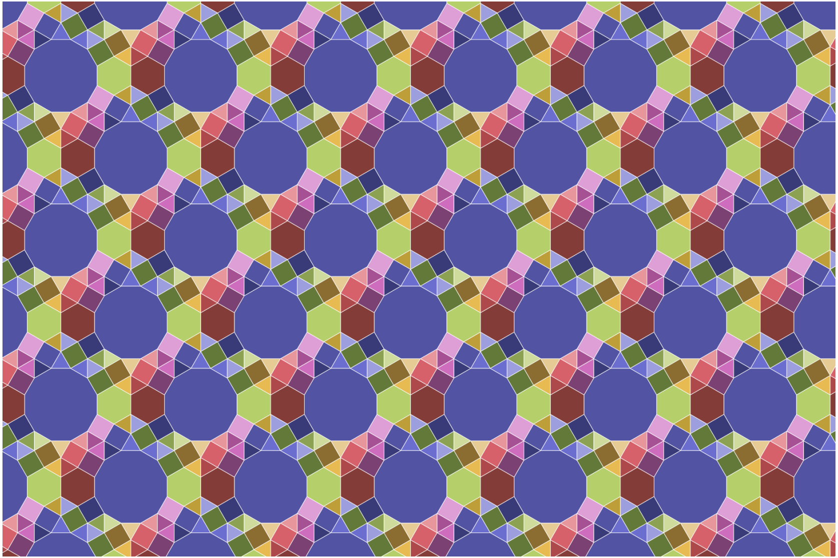
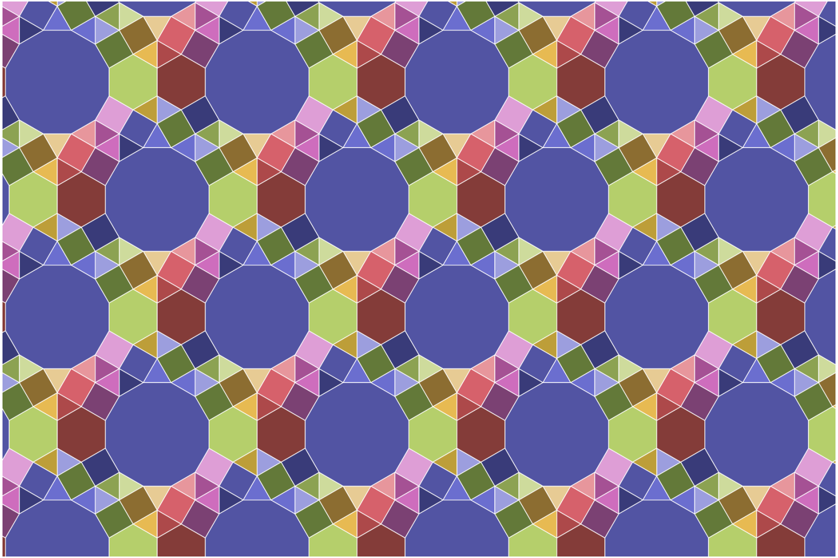


image by C. Kaplan

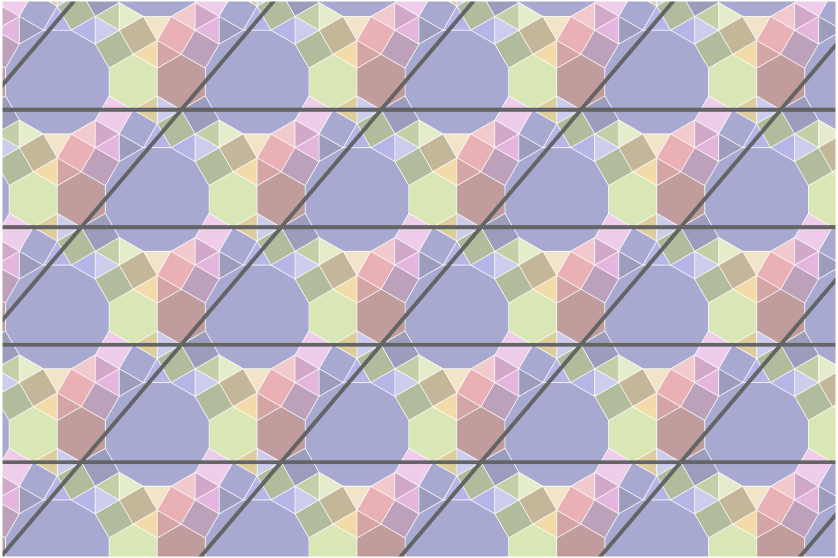
Goal: represent and synthesize complex k-regular tilings



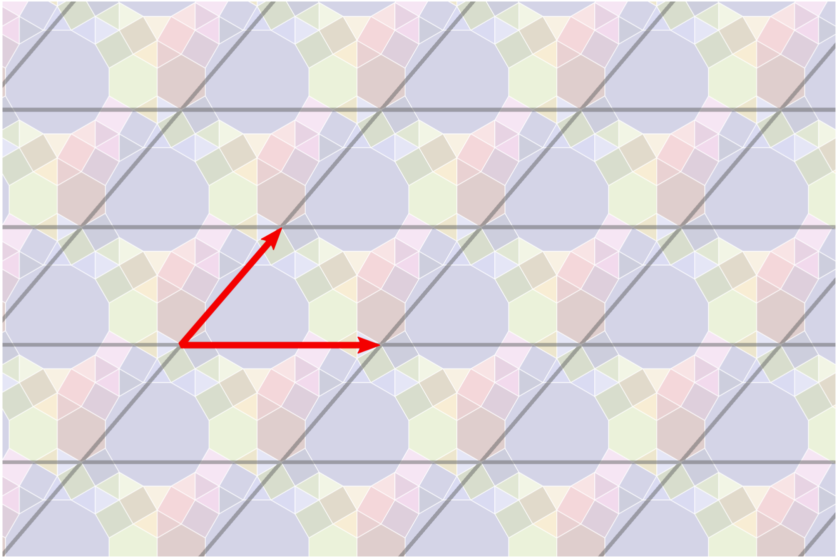
Understanding tilings: many symmetries



Understanding tilings: translation symmetries (group p1)

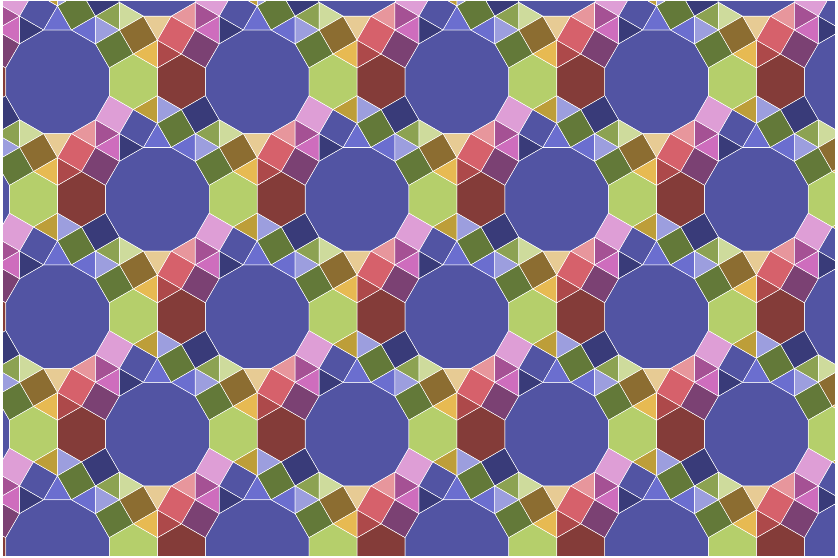


Understanding tilings: fundamental domain



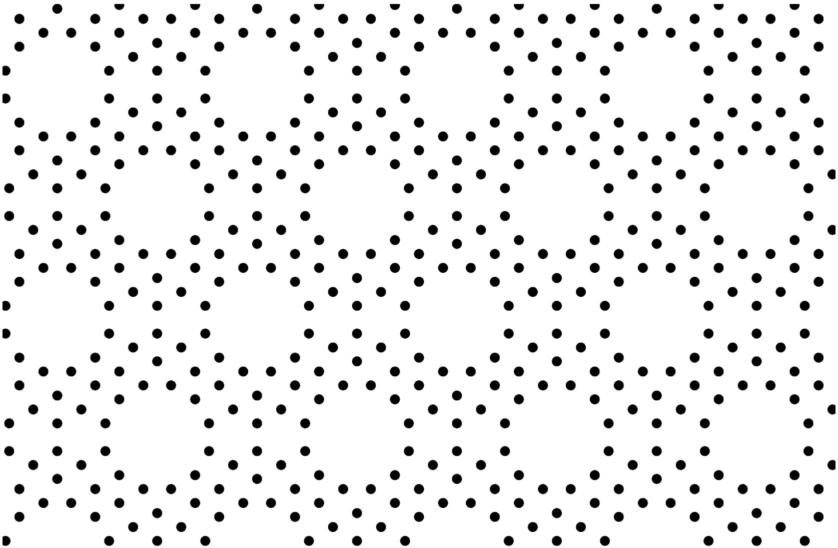
Regular systems of points

Hilbert & Cohn-Vossen (1952)

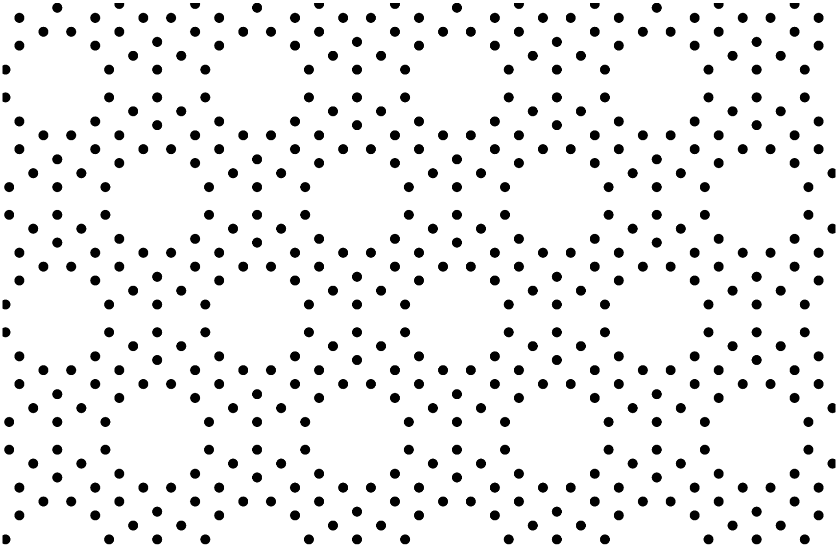


Regular systems of points

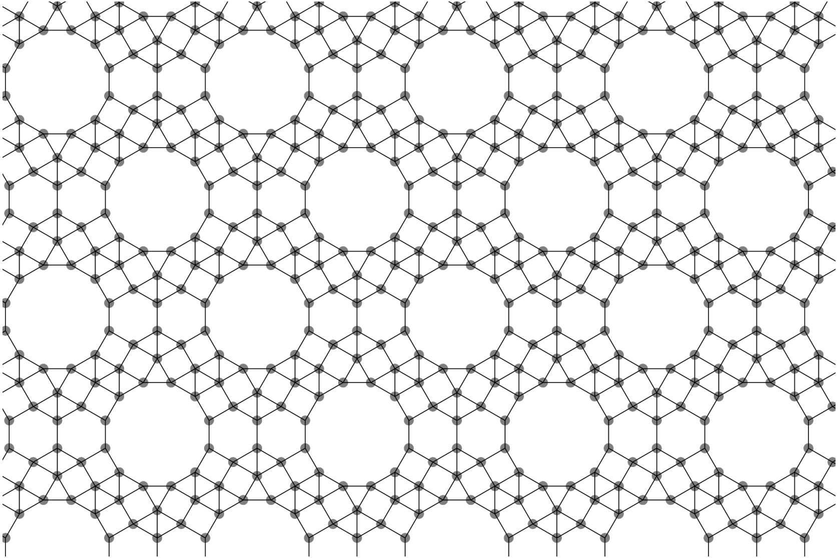
Hilbert & Cohn-Vossen (1952)



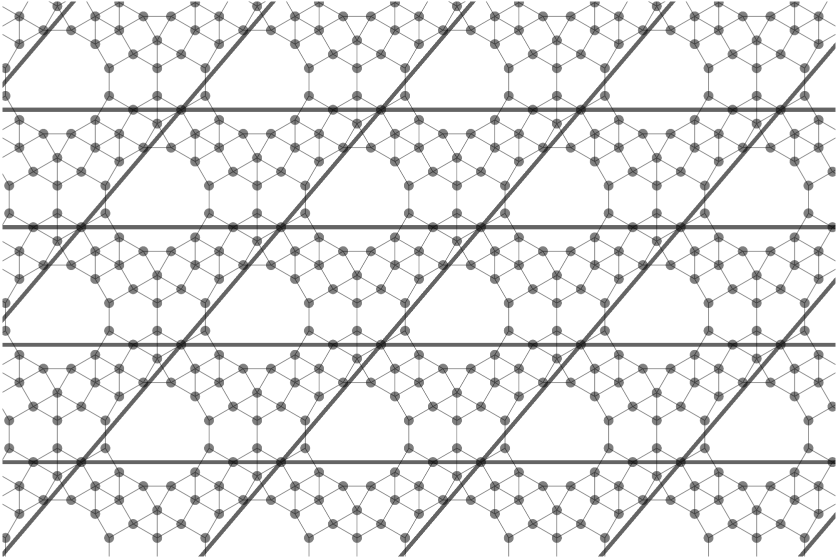
Reconstruct tiling from vertices



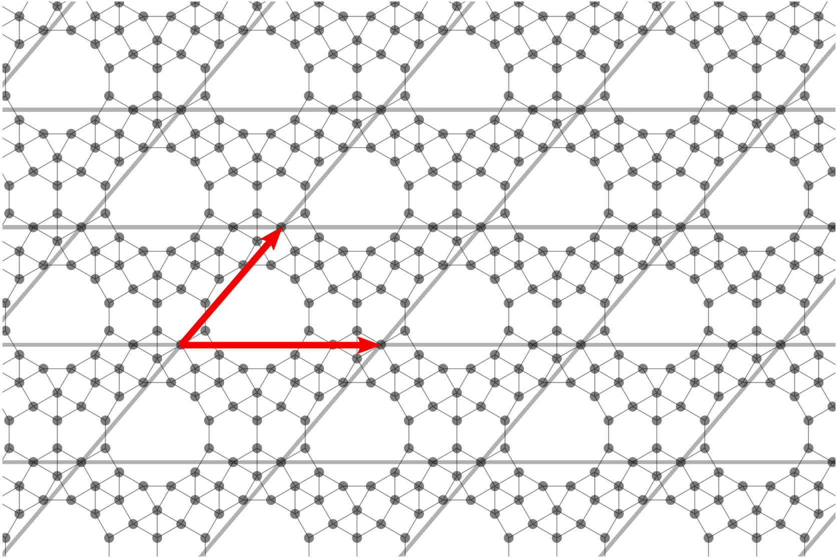
Reconstruct tiling from vertices: edges



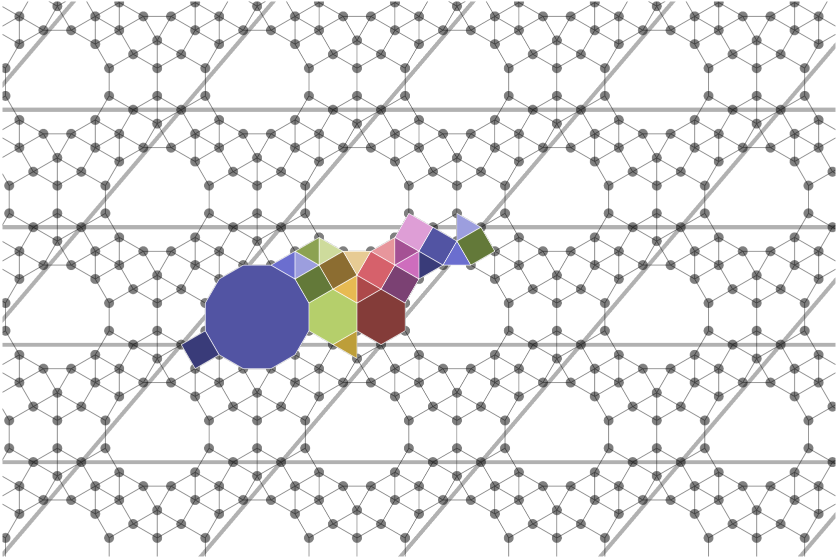
Reconstruct tiling from vertices: translation grid



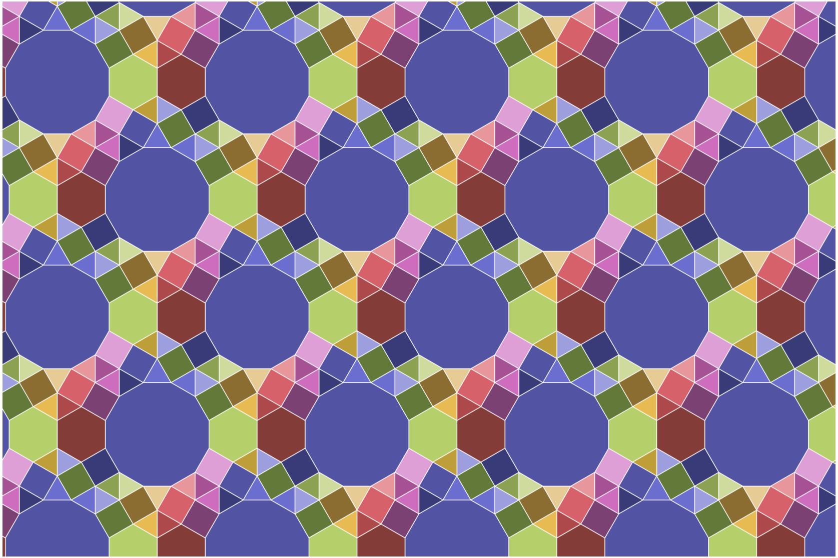
Reconstruct tiling from vertices: fundamental domain



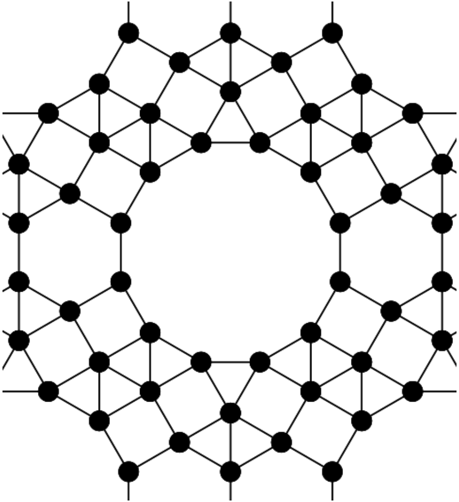
Reconstruct tiling from vertices: patch



Reconstruct tiling from vertices: full tiling



Edges aligned to a few basic directions

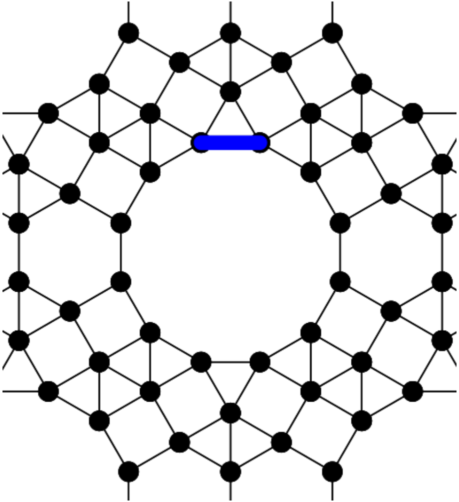


roots of unity

$$\pm\omega_n,\ \pm\overline{\omega}_n \qquad \omega_n = e^{\frac{2\pi i}{n}}$$

$$n \in \{1, 2, 3, 4, 6, 8, 12\}$$

Edges aligned to a few basic directions



roots of unity

$$\pm\omega_n,\ \pm\overline{\omega}_n \qquad \omega_n = e^{\frac{2\pi i}{n}}$$

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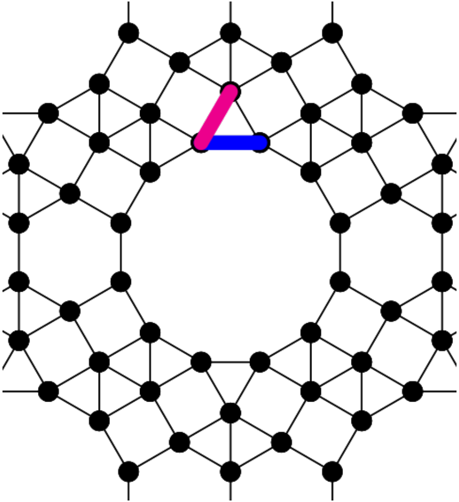
 $+\omega_1$

Edges aligned to a few basic directions



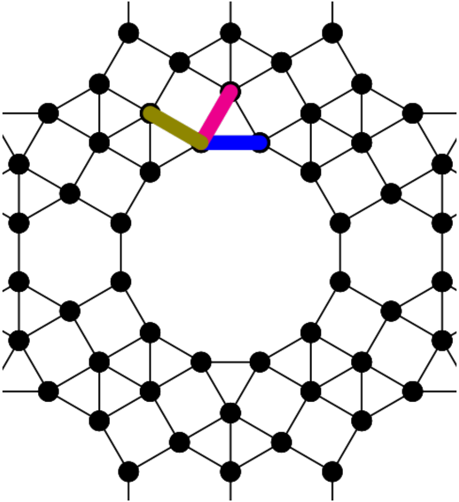
roots of unity

$$\pm\omega_n,\ \pm\overline{\omega}_n \qquad \omega_n = e^{\frac{2\pi i}{n}}$$

$$n \in \{1, 2, 3, 4, 6, 8, 12\}$$

- $+\omega_1$
- $+\omega_6$

Edges aligned to a few basic directions



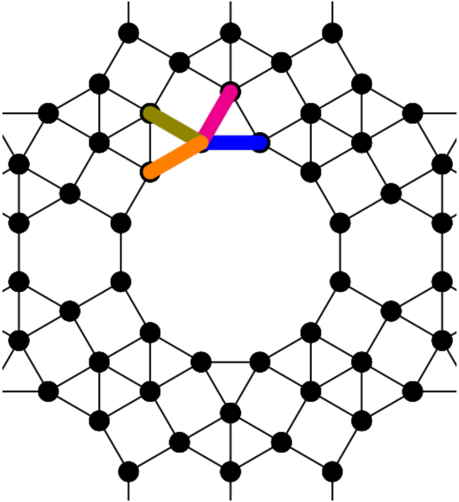
roots of unity

$$\pm\omega_n,\ \pm\overline{\omega}_n \qquad \omega_n = e^{\frac{2\pi i}{n}}$$

$$n \in \{1, 2, 3, 4, 6, 8, 12\}$$

- $+\omega_1$
- $+\omega_6$
- $-\overline{\omega}_{12}$

Edges aligned to a few basic directions



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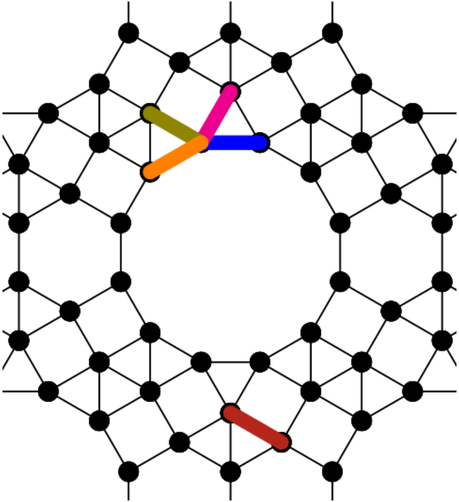
Edges aligned to a few basic directions



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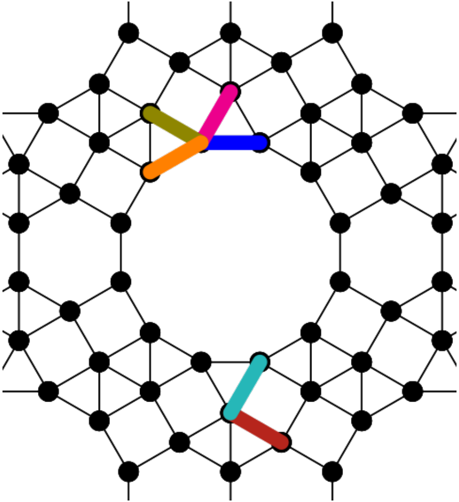
$$n \in \{1, 2, 3, 4, 6, 8, 12\}$$

- $+\omega_1$
- $+\omega_6$
- $-\overline{\omega}_{12}$
- $-\omega_{12}$
- $+\overline{\omega}_{12}$

Edges aligned to a few basic directions



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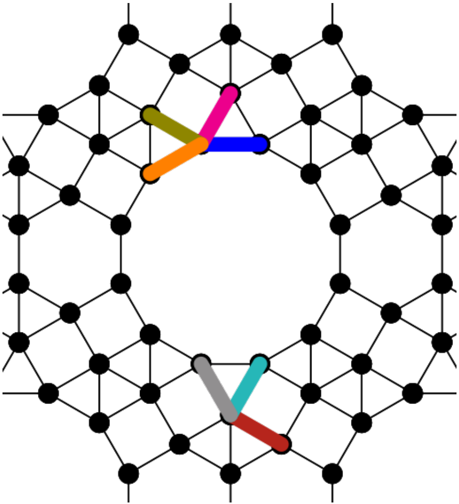
- $+\omega_1$
- $+\omega_6$
- $-\overline{\omega}_{12}$
- $-\omega_{12}$
- $+\overline{\omega}_{12}$
- $+\omega_6$

Edges aligned to a few basic directions



roots of unity

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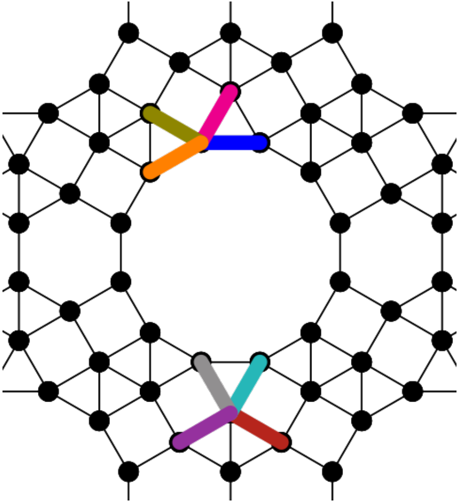
- $+\omega_1$
- $+\omega_6$
- $-\overline{\omega}_{12}$
- $-\omega_{12}$
- $+\overline{\omega}_{12}$
- $+\omega_6$
- $-\overline{\omega}_6$

Edges aligned to a few basic directions



roots of unity

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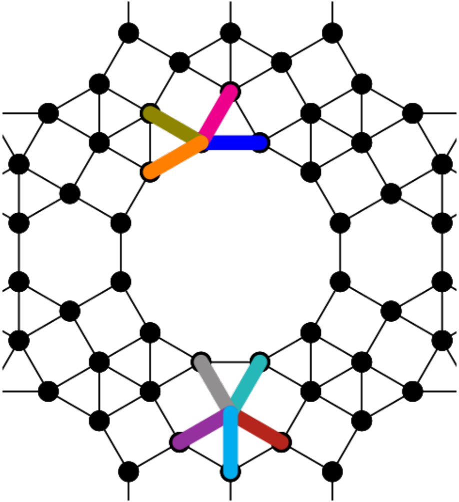
$$n \in \{1, 2, 3, 4, 6, 8, 12\}$$

- $+\omega_1$
- $+\omega_6$
- $-\overline{\omega}_{12}$
- $-\omega_{12}$
- $+\overline{\omega}_{12}$
- $+\omega_6$
- $-\overline{\omega}_6$
- $-\omega_{12}$

Edges aligned to a few basic directions



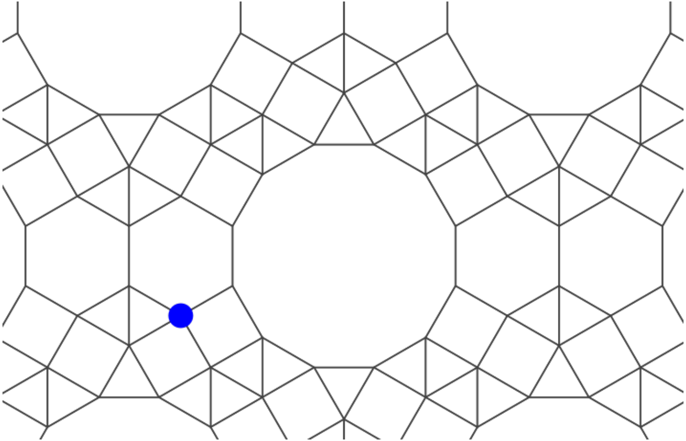
roots of unity

$$\pm\omega_n, \ \pm\overline{\omega}_n \qquad \omega_n = e^{\frac{2\pi i}{n}}$$

$$n \in \{1, 2, 3, 4, 6, 8, 12\}$$

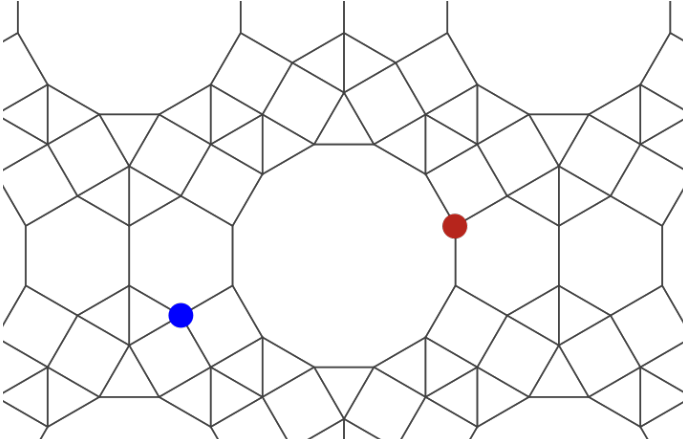
- $+\omega_1$
- $+\omega_6$
- $-\overline{\omega}_{12}$
- $-\omega_{12}$
- $+\overline{\omega}_{12}$
- $+\omega_6$
- $-\overline{\omega}_6$
- $-\omega_{12}$
- $-\omega_4$

Vertices as integer linear combinations of basic directions



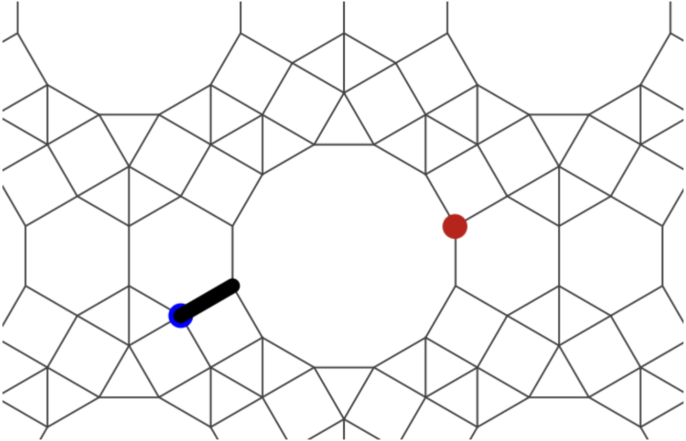
O

Vertices as integer linear combinations of basic directions



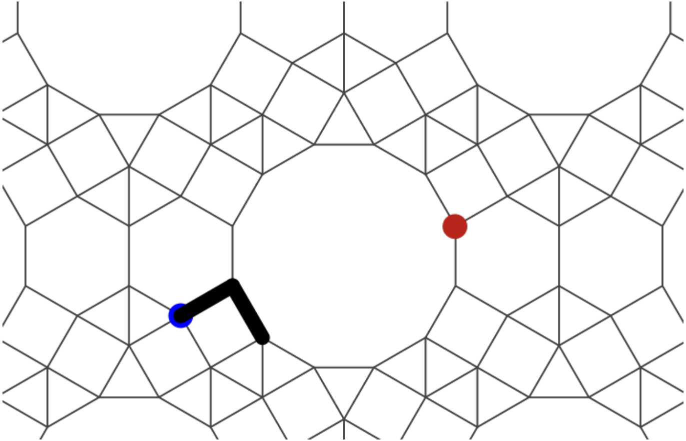
O

Vertices as integer linear combinations of basic directions



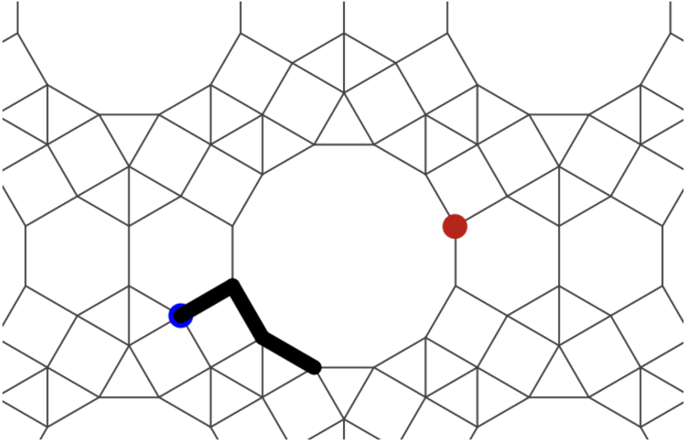
$O + \omega_{12}$

Vertices as integer linear combinations of basic directions



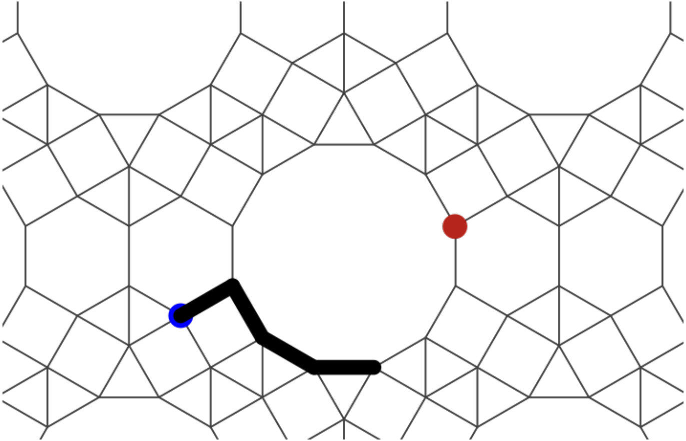
$O + \omega_{12} + \overline{\omega}_6$

Vertices as integer linear combinations of basic directions



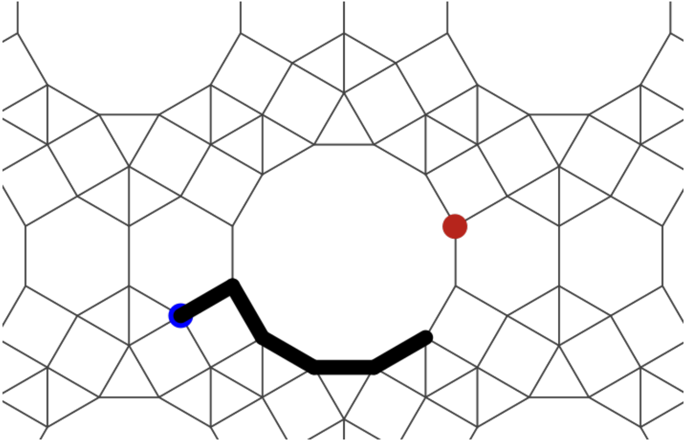
$\color{blue}{O} + \omega_{12} + \overline{\omega}_6 + \overline{\omega}_{12}$

Vertices as integer linear combinations of basic directions



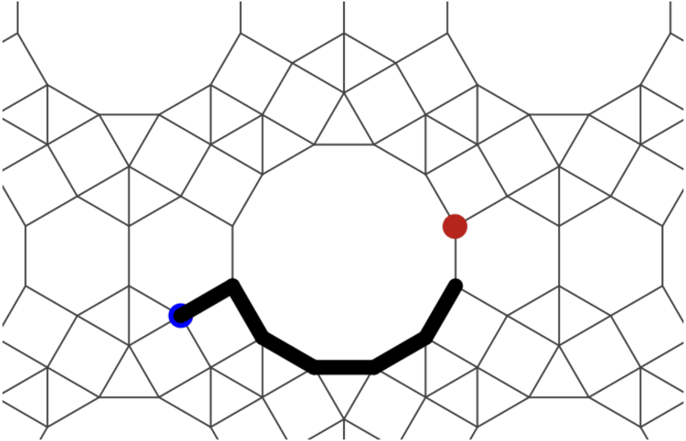
$O + \omega_{12} + \overline{\omega}_6 + \overline{\omega}_{12} + \omega_1$

Vertices as integer linear combinations of basic directions



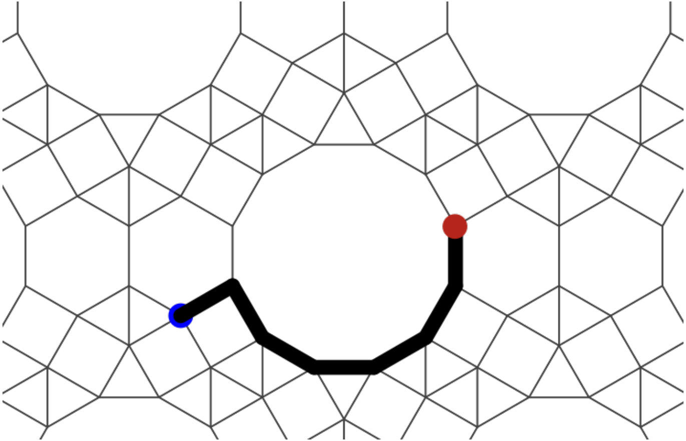
$O + \omega_{12} + \overline{\omega}_6 + \overline{\omega}_{12} + \omega_1 + \omega_{12}$

Vertices as integer linear combinations of basic directions



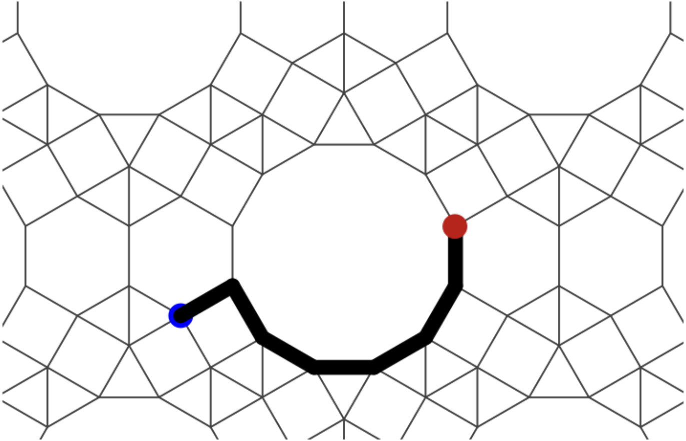
$$\color{blue}{O} + \omega_{12} + \overline{\omega}_6 + \overline{\omega}_{12} + \omega_1 + \omega_{12} + \omega_6$$

Vertices as integer linear combinations of basic directions



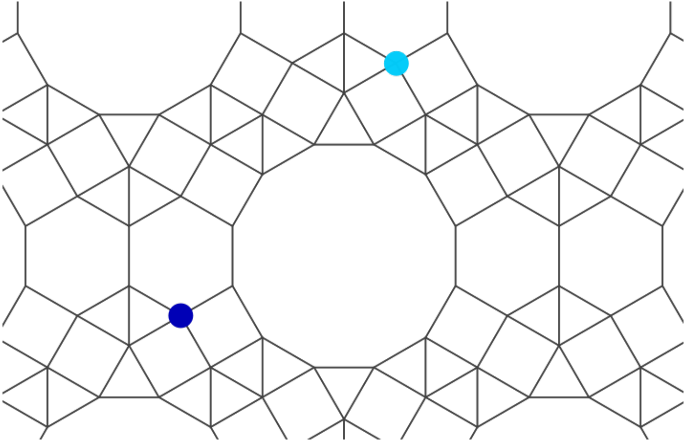
$$O + \omega_{12} + \overline{\omega}_6 + \overline{\omega}_{12} + \omega_1 + \omega_{12} + \omega_6 + \omega_4$$

Vertices as integer linear combinations of basic directions



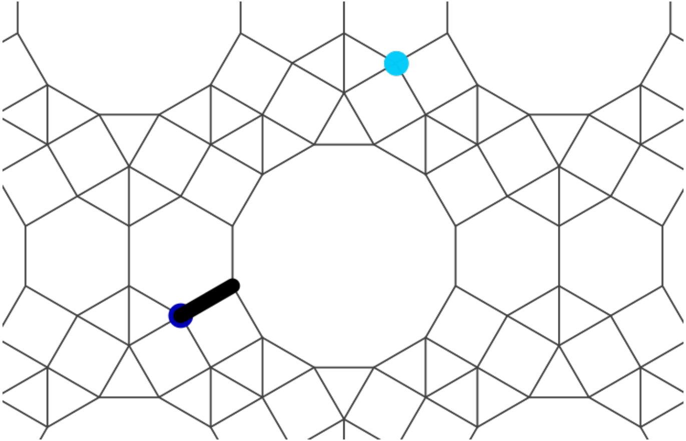
$$\color{blue}{O} + \omega_{12} + \overline{\omega}_6 + \overline{\omega}_{12} + \omega_1 + \omega_{12} + \omega_6 + \omega_4 = \color{red}{v}$$

Translations as integer linear combinations of basic directions



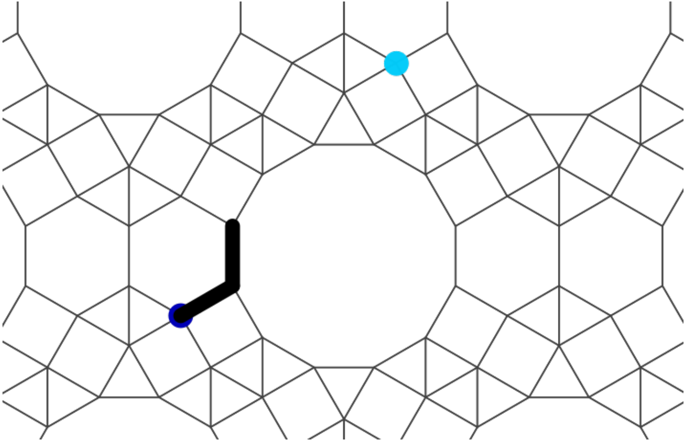
O

Translations as integer linear combinations of basic directions



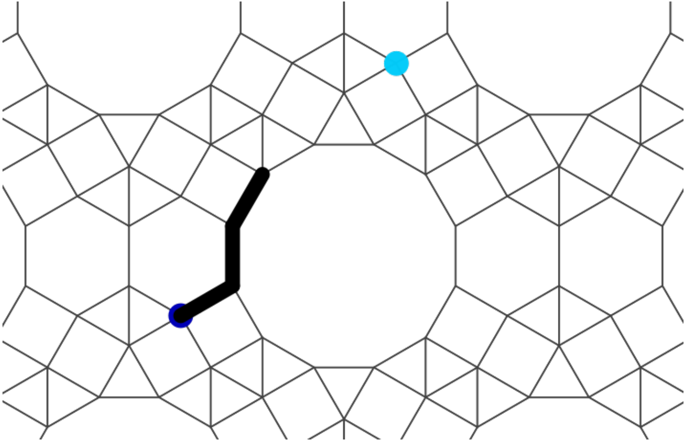
$O + \omega_{12}$

Translations as integer linear combinations of basic directions



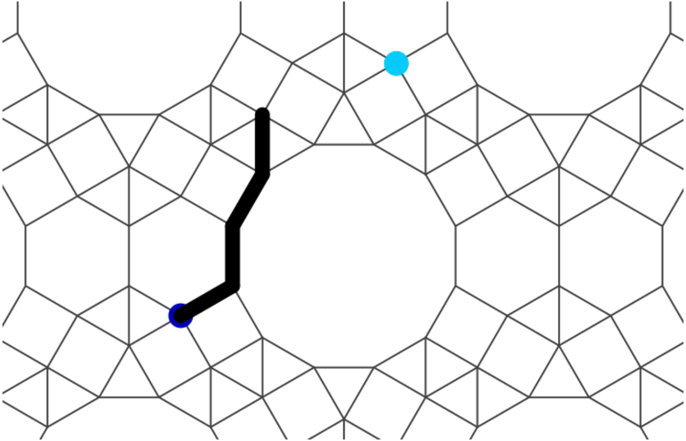
$O + \omega_{12} + \omega_4$

Translations as integer linear combinations of basic directions



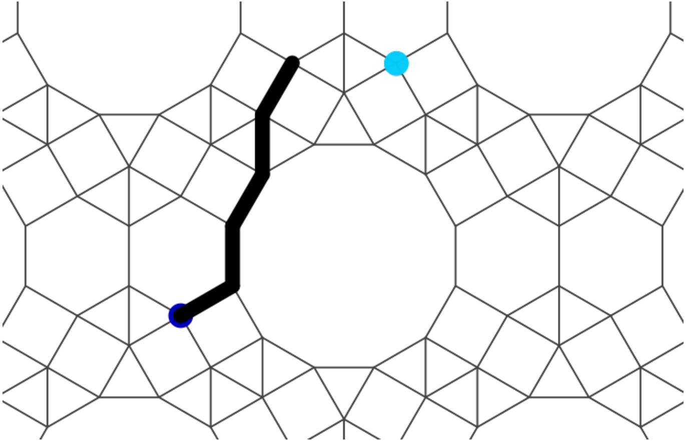
$O + \omega_{12} + \omega_4 + \omega_6$

Translations as integer linear combinations of basic directions



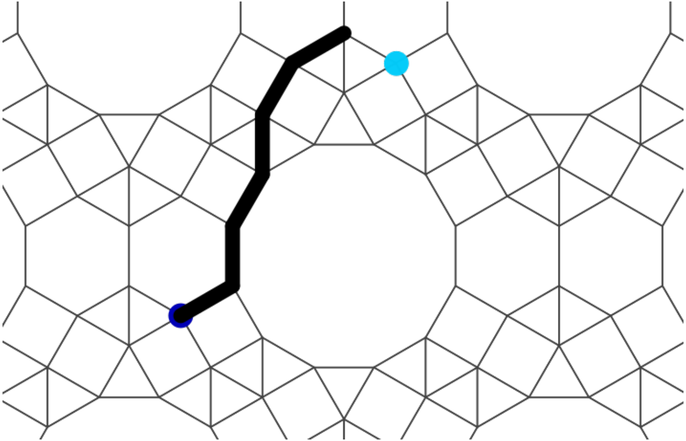
$O + \omega_{12} + \omega_4 + \omega_6 + \omega_4$

Translations as integer linear combinations of basic directions



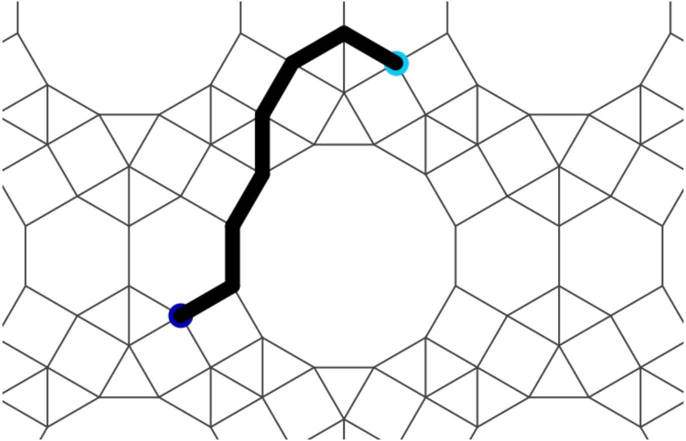
$O + \omega_{12} + \omega_4 + \omega_6 + \omega_4 + \omega_6$

Translations as integer linear combinations of basic directions



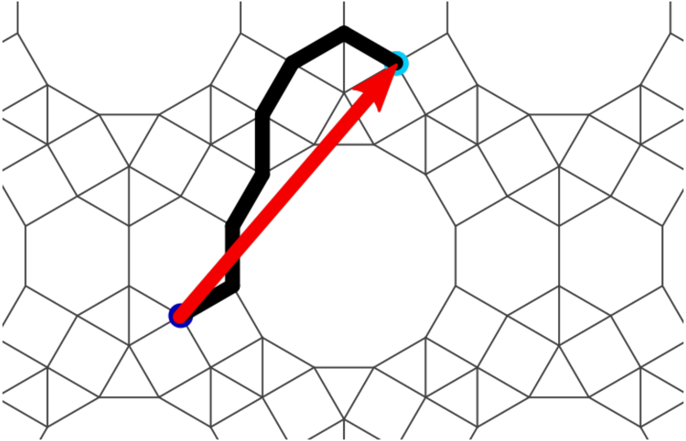
$$\color{blue}{O} + \omega_{12} + \omega_4 + \omega_6 + \omega_4 + \omega_6 + \omega_{12}$$

Translations as integer linear combinations of basic directions



$$O + \omega_{12} + \omega_4 + \omega_6 + \omega_4 + \omega_6 + \omega_{12} + \overline{\omega}_{12}$$

Translations as integer linear combinations of basic directions



$$\color{blue}{O} + \omega_{12} + \omega_4 + \omega_6 + \omega_4 + \omega_6 + \omega_{12} + \overline{\omega}_{12} = \color{red}{t}$$

Tiling symbols

Each tiling is represented by:

- two translation vectors
  define the fundamental region
- set of seeds
  vertices inside fundamental region
- translation vectors and seeds expressed as integer linear combinations of basic directions

Basic directions can be simplified:

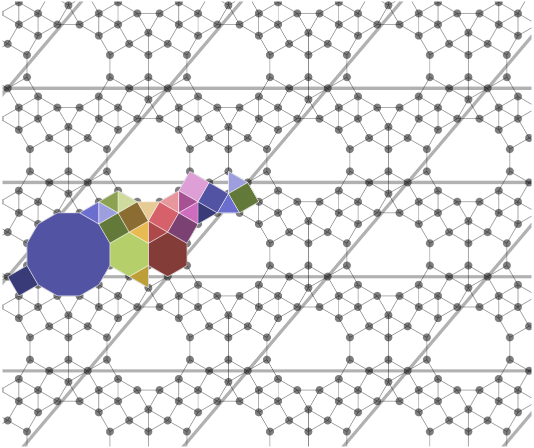
$$\omega_2 + \omega_1 = 0$$
$$\omega_3 + \overline{\omega}_6 = 0$$
$$\omega_2 + \omega_6 + \overline{\omega}_6 = 0$$
$$\omega_4 - \omega_{12} + \overline{\omega}_{12} = 0$$

$$\omega_1 = 1, \quad \omega_2 = -1, \quad \omega_4 = i$$

At most four basic directions needed

Unique representation

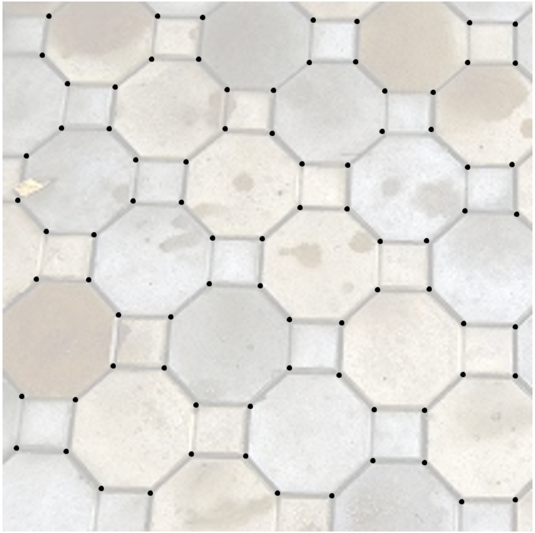
Tiling symbols



basic directions	$\Omega = \{\omega_1, \omega_4, \omega_6, \omega_{12}\}$
translation vectors	$t_1 = [0, 1, 2, 3]$ $t_2 = [2, -3, 0, 6]$
seeds	$S_1 = [0, 0, 1, 2]$ $S_2 = [0, 0, 1, 3]$ $S_3 = [1, 0, 0, 1]$ $\dots$ $S_{24} = [2, 3, 1, 1]$

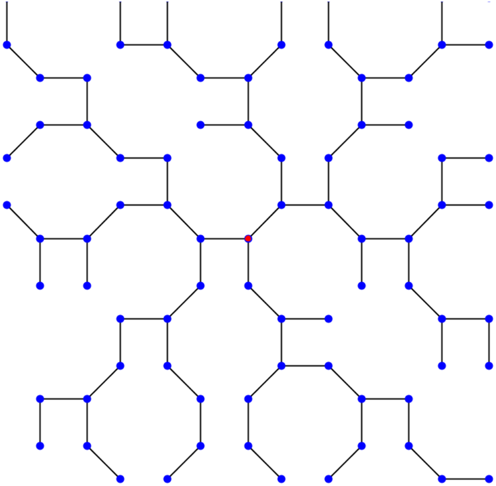
Assisted acquisition from photographs

Assisted acquisition from photographs: user selects vertices



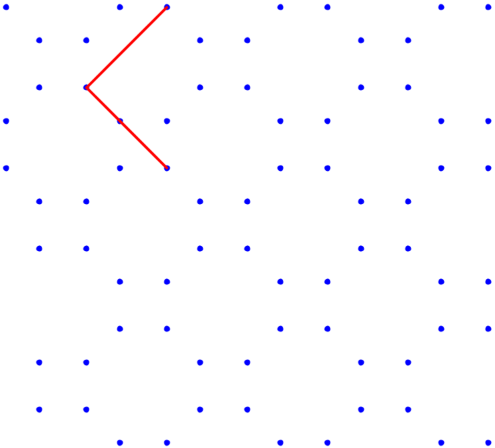
Assisted acquisition from photographs: user selects vertices

Assisted acquisition from photographs: connect vertices with basic directions



$$\Omega = \{\omega_1, \omega_4, \omega_8, \overline{\omega}_8\}$$

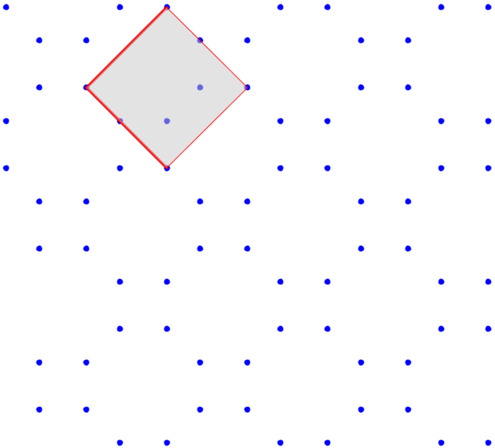
Assisted acquisition from photographs: find translations by maximizing overlap



$$\Omega = \{\omega_1, \omega_4, \omega_8, \overline{\omega}_8\}$$

$$t_1 = [1, 1, 1, 0]$$
$$t_2 = [1, -1, 0, 1]$$

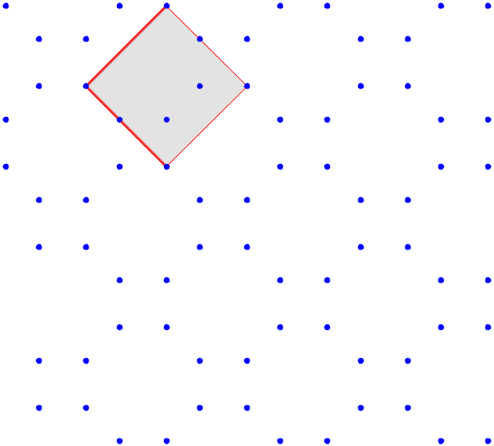
Assisted acquisition from photographs: fundamental region



$$\Omega = \{\omega_1, \omega_4, \omega_8, \overline{\omega}_8\}$$

$$t_1 = [1, 1, 1, 0]$$
$$t_2 = [1, -1, 0, 1]$$

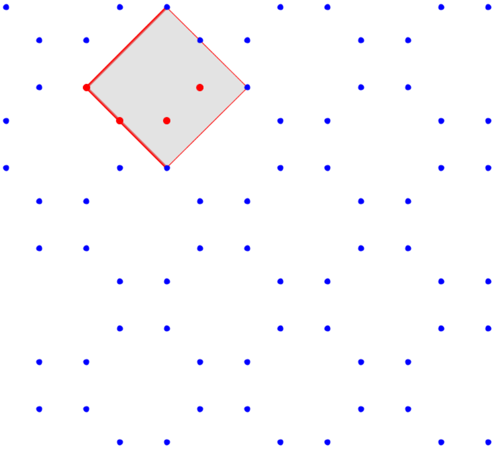
Assisted acquisition from photographs: fundamental region



$$\Omega = \{\omega_1, \omega_4, \omega_8, \overline{\omega}_8\}$$

$$t_1 = [1, 1, 1, 0]$$
$$t_2 = [1, -1, 0, 1]$$

Assisted acquisition from photographs: seeds

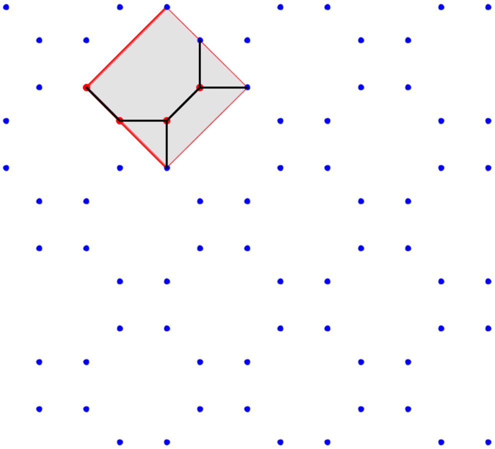


$$\Omega = \{\omega_1, \omega_4, \omega_8, \overline{\omega}_8\}$$

$$t_1 = [1, 1, 1, 0]$$
$$t_2 = [1, -1, 0, 1]$$

$$S_1 = [0, 0, 0, 0]$$
$$S_2 = [0, 0, 0, 1]$$
$$S_3 = [1, 0, 0, 1]$$
$$S_4 = [1, 0, 1, 1]$$

Assisted acquisition from photographs: patch

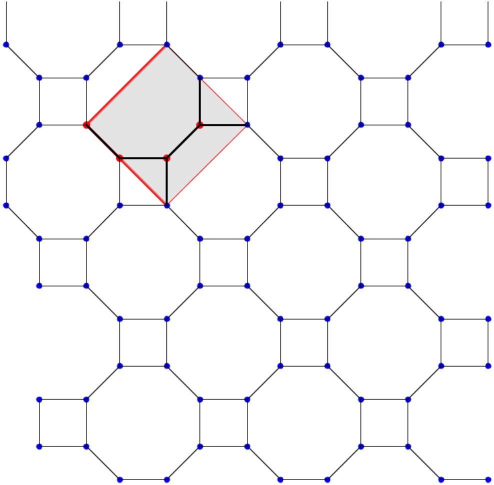


$$\Omega = \{\omega_1, \omega_4, \omega_8, \overline{\omega}_8\}$$

$$t_1 = [1, 1, 1, 0]$$
$$t_2 = [1, -1, 0, 1]$$

$$S_1 = [0, 0, 0, 0]$$
$$S_2 = [0, 0, 0, 1]$$
$$S_3 = [1, 0, 0, 1]$$
$$S_4 = [1, 0, 1, 1]$$

Assisted acquisition from photographs: tiling

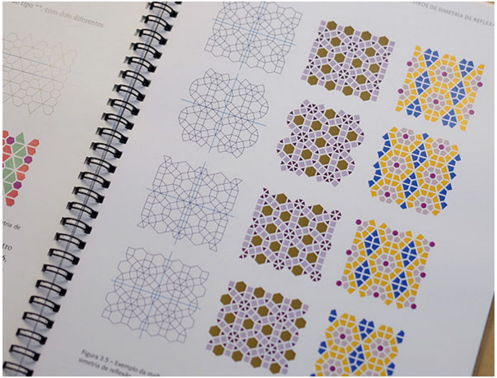


$$\Omega = \{\omega_1, \omega_4, \omega_8, \overline{\omega}_8\}$$

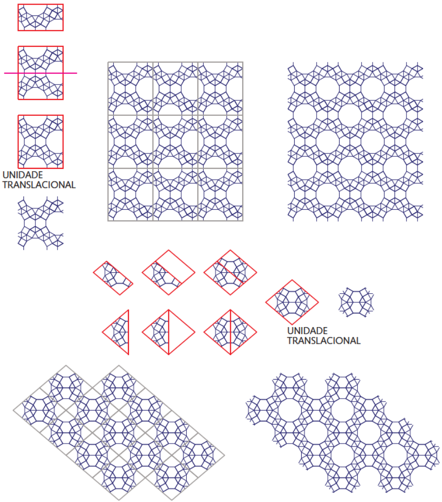
$$t_1 = [1, 1, 1, 0]$$
$$t_2 = [1, -1, 0, 1]$$

$$S_1 = [0, 0, 0, 0]$$
$$S_2 = [0, 0, 0, 1]$$
$$S_3 = [1, 0, 0, 1]$$
$$S_4 = [1, 0, 1, 1]$$

Book & catalogue: 200+ Arquimedean tilings

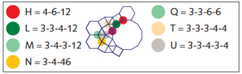


"Sobre malhas arquimedianas", Ricardo Sá e Asla Medeiros e Sá, 2017

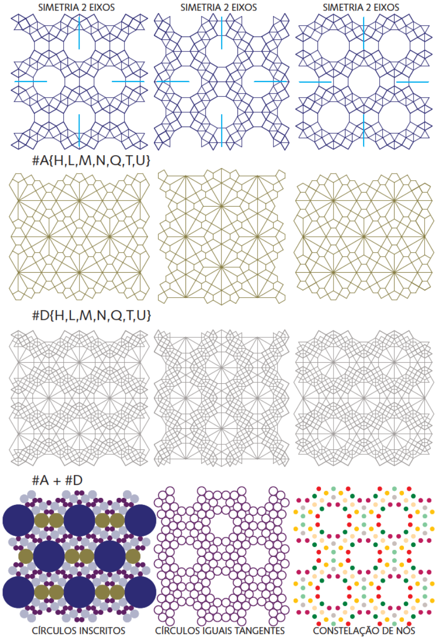

UNIDADE
TRANSLACIONAL
UNIDADE
TRANSLACIONAL

#A{H,L,M,N,Q,T,U}

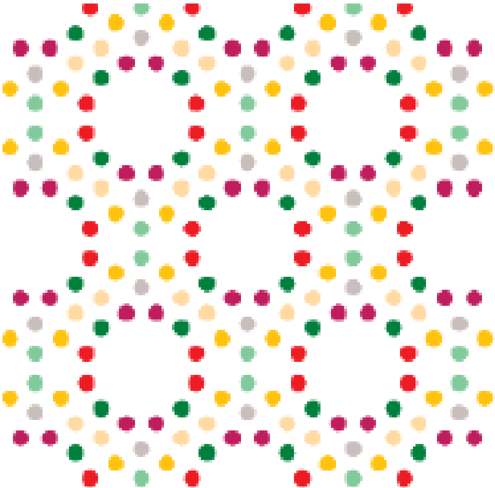

FRAGMENTO
FUNDAMENTAL


H = 4-6-12
L = 3-3-4-12
M = 3-4-3-12
N = 3-4-46
Q = 3-3-6-6
T = 3-3-3-4-4
U = 3-3-4-3-4

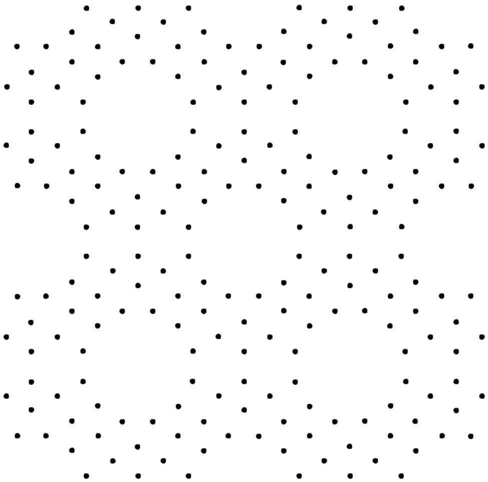

POLÍGONOS DUAIS


SIMETRIA 2 EIXOS
SIMETRIA 2 EIXOS
SIMETRIA 2 EIXOS
#A{H,L,M,N,Q,T,U}
#D{H,L,M,N,Q,T,U}
#A + #D
CÍRCULOS INSCRITOS
CÍRCULOS IGUAIS TANGENTES
CONSTELAÇÃO DE NÓS

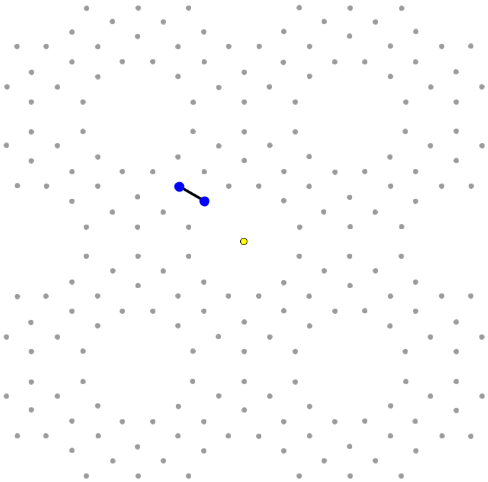
Automatic acquisition from tiny point cloud images



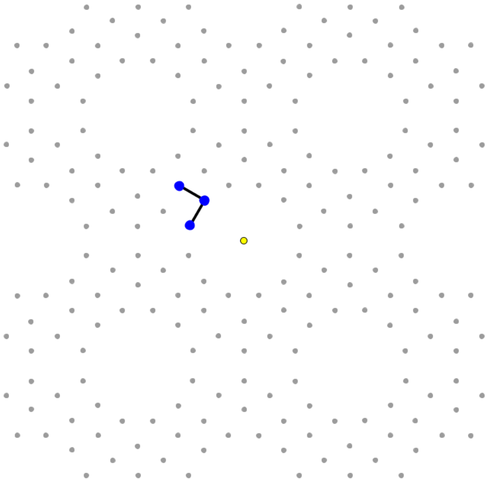
Automatic acquisition from tiny point cloud images



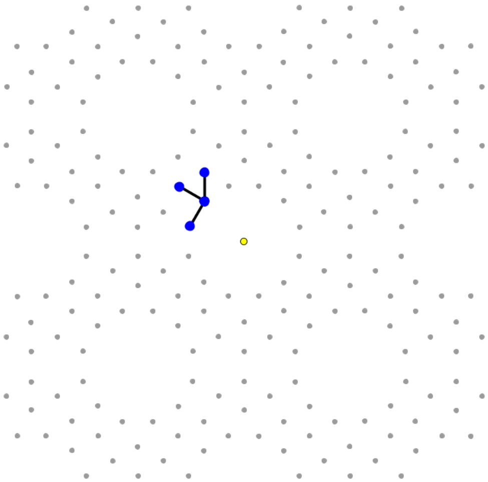
Automatic acquisition from tiny point cloud images



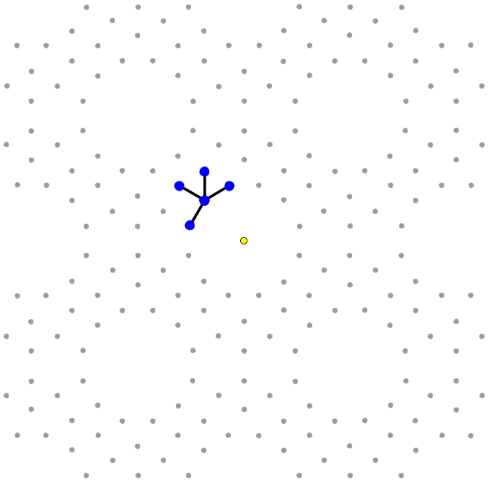
Automatic acquisition from tiny point cloud images



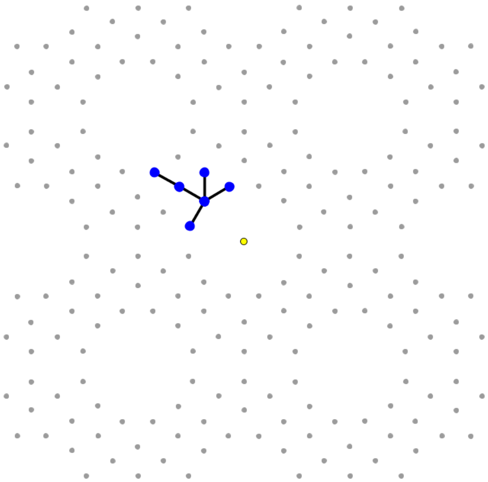
Automatic acquisition from tiny point cloud images



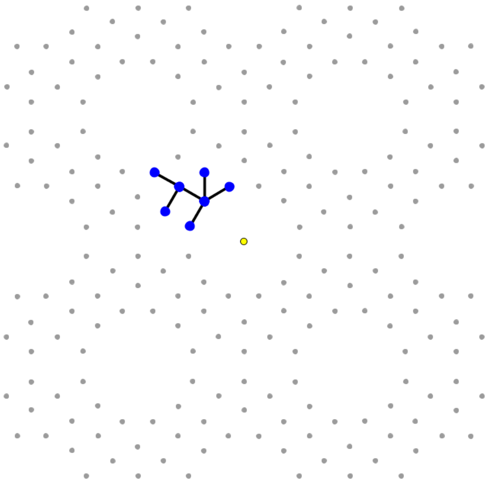
Automatic acquisition from tiny point cloud images



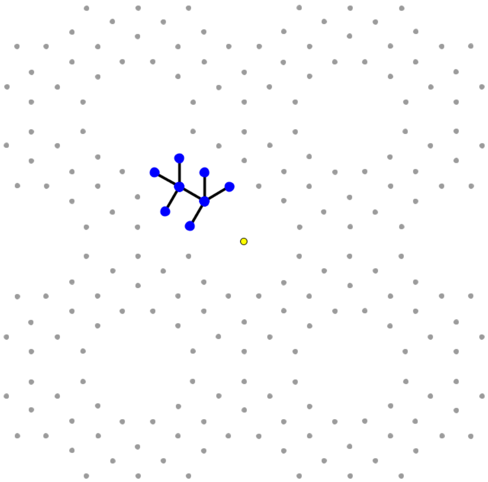
Automatic acquisition from tiny point cloud images



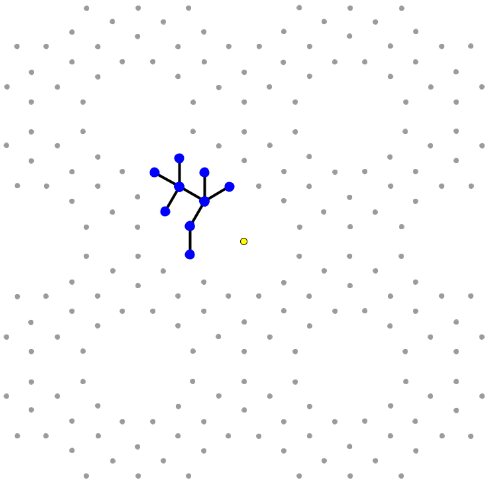
Automatic acquisition from tiny point cloud images



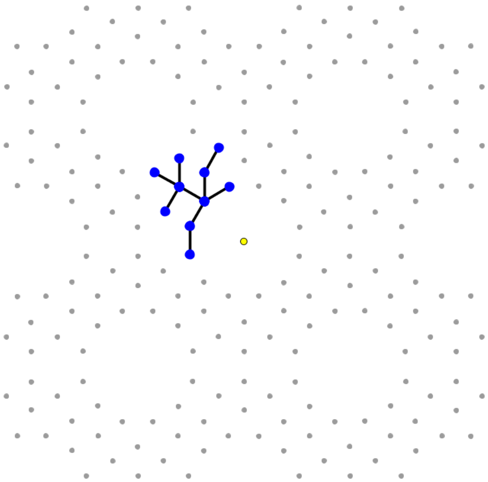
Automatic acquisition from tiny point cloud images



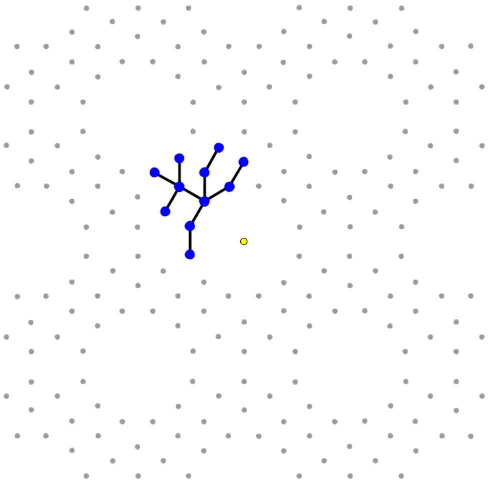
Automatic acquisition from tiny point cloud images



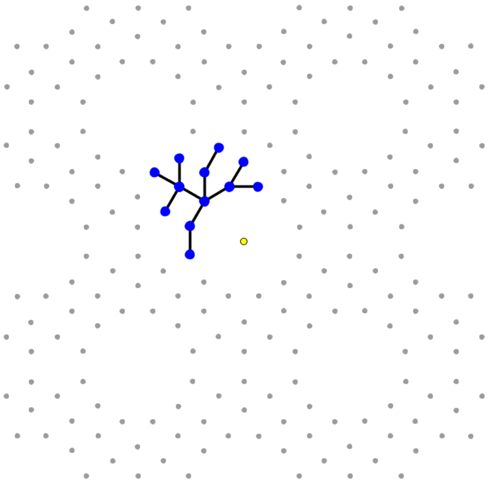
Automatic acquisition from tiny point cloud images



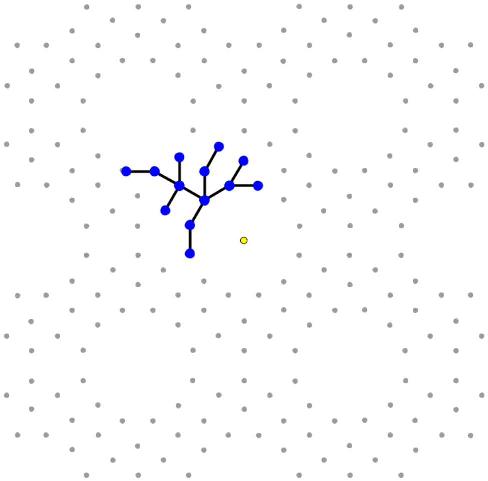
Automatic acquisition from tiny point cloud images



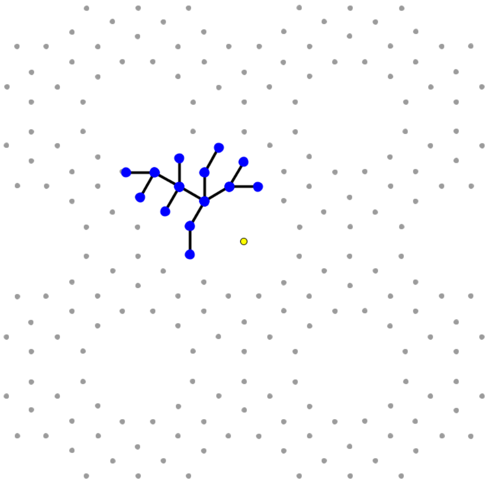
Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



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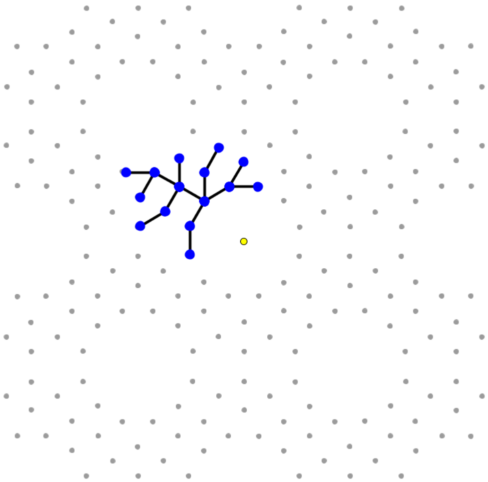
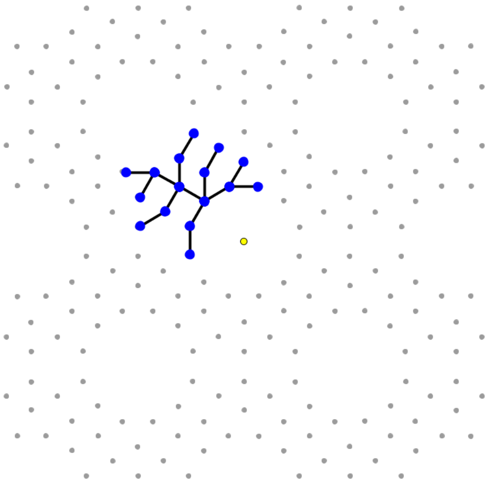


Automatic acquisition from tiny point cloud images



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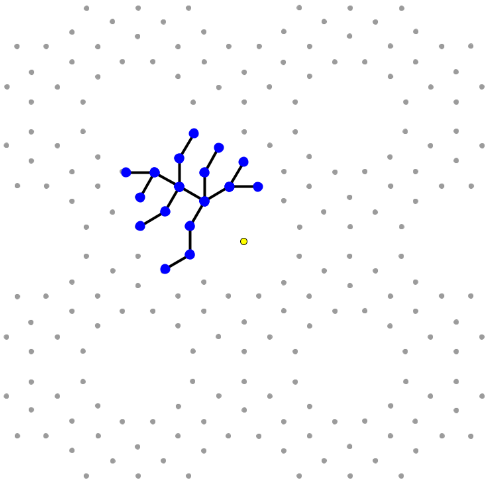
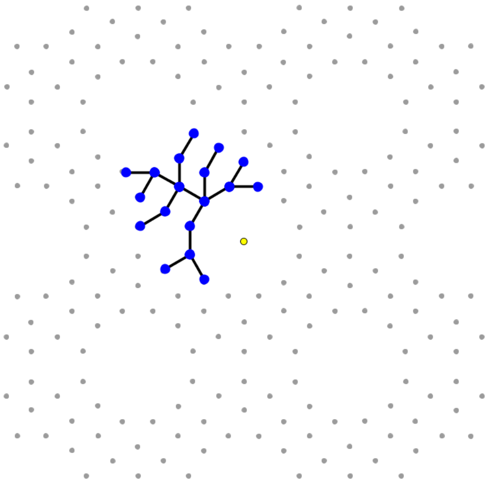
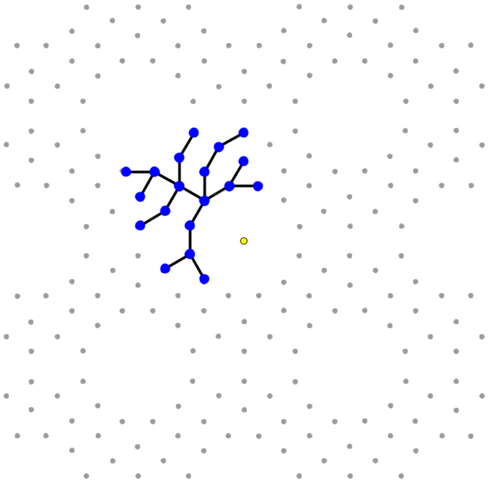


Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



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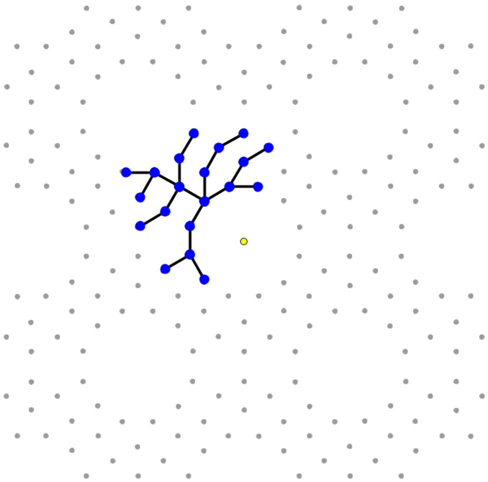
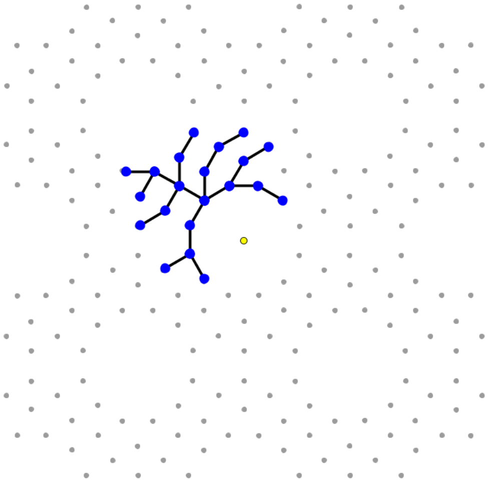
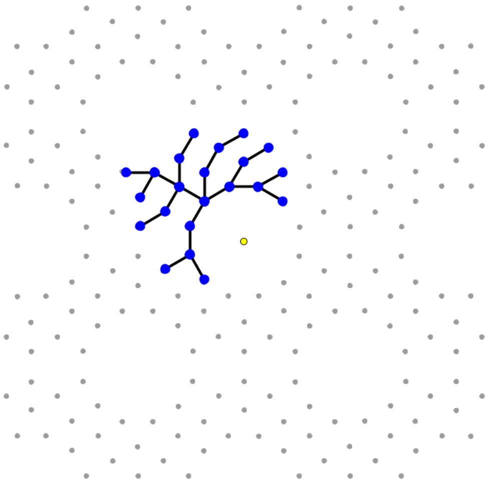


Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



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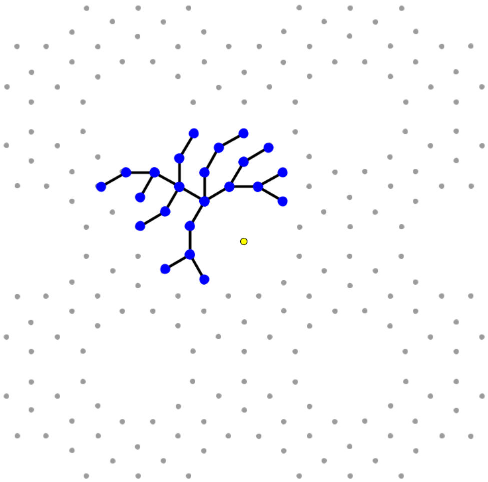
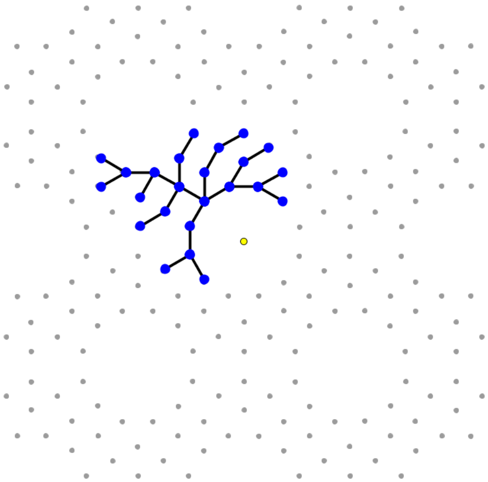


Automatic acquisition from tiny point cloud images



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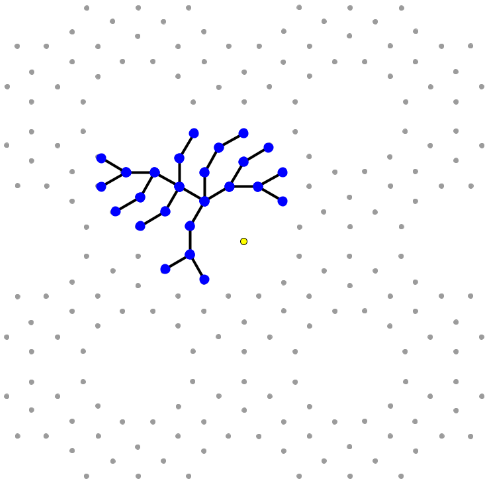

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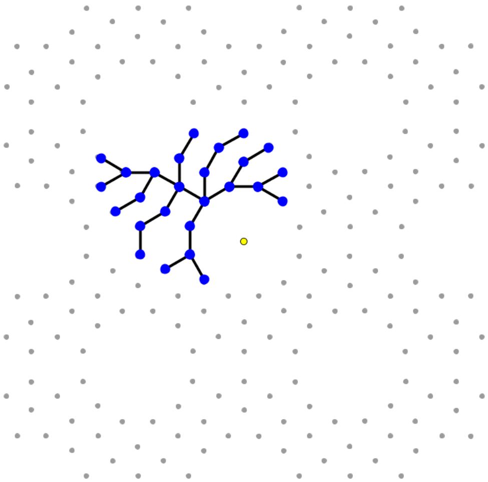
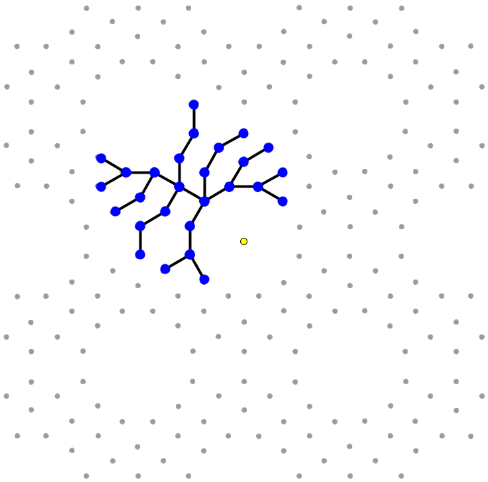
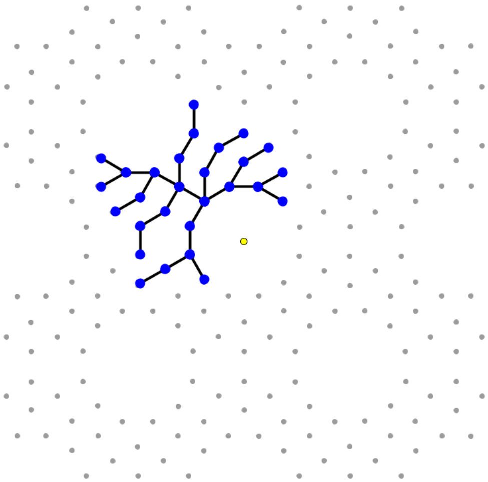


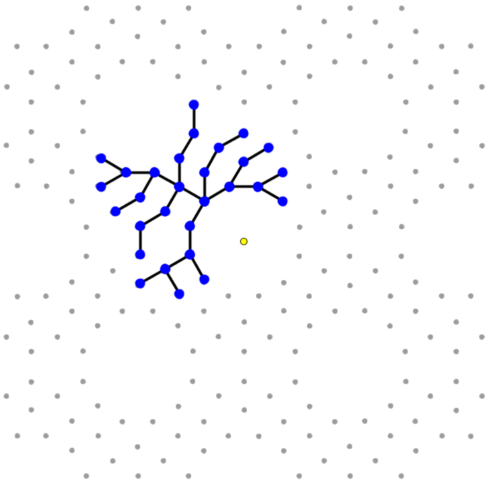
Automatic acquisition from tiny point cloud images



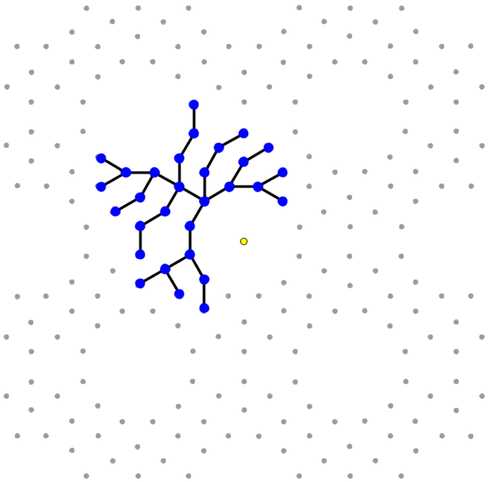
Automatic acquisition from tiny point cloud images



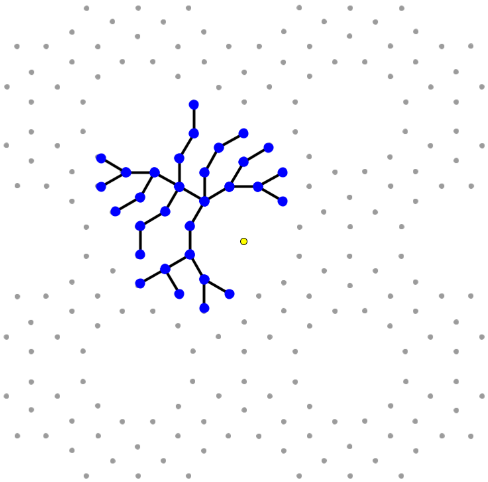
Automatic acquisition from tiny point cloud images



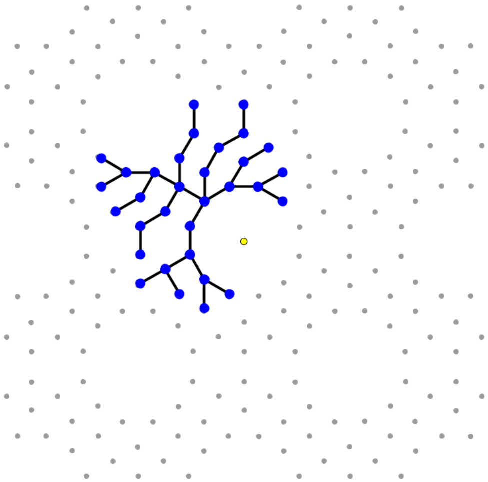
Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



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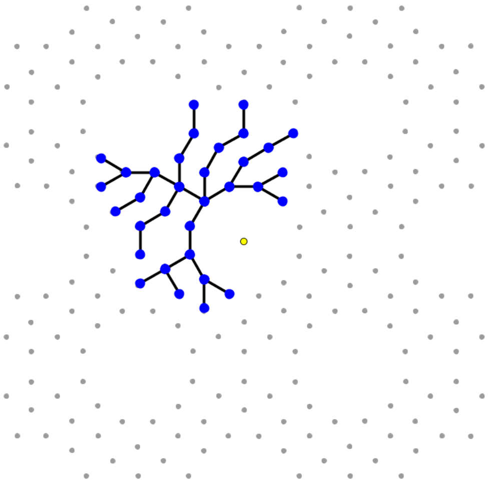

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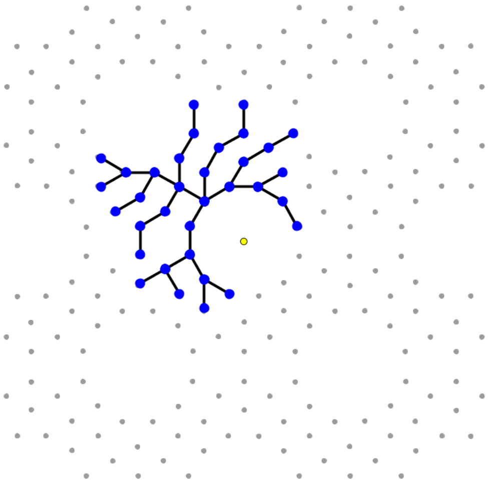
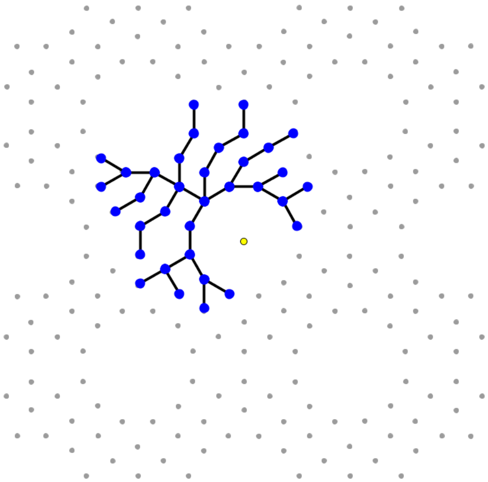
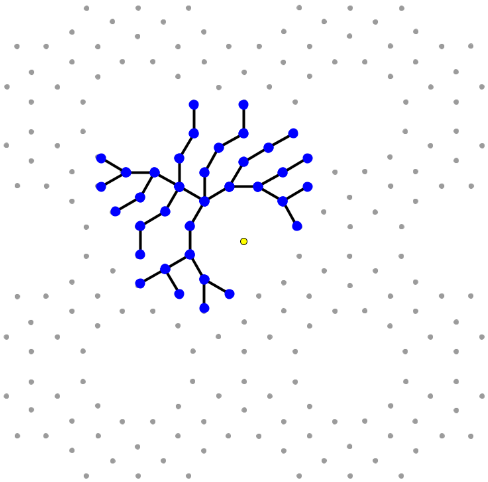


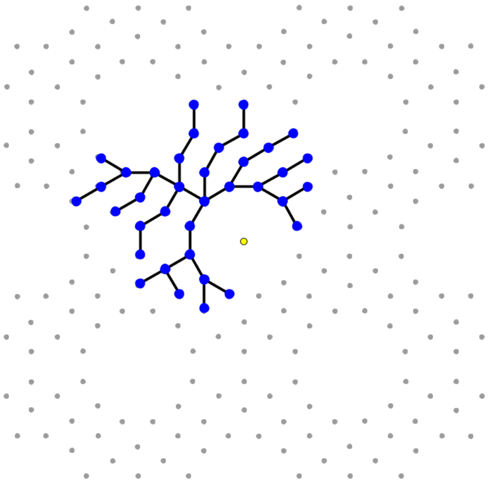
Automatic acquisition from tiny point cloud images



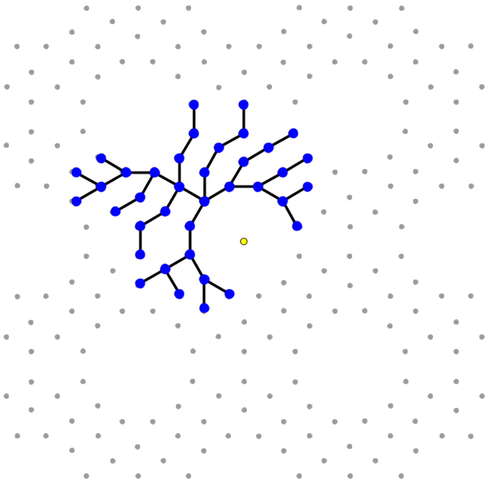
Automatic acquisition from tiny point cloud images



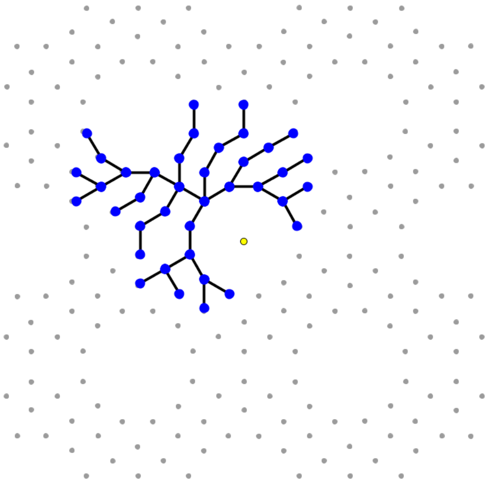
Automatic acquisition from tiny point cloud images



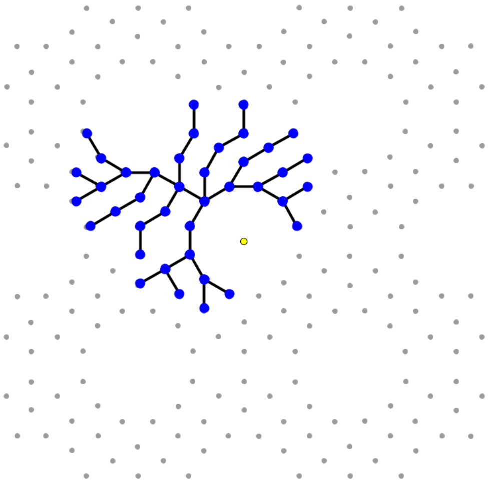
Automatic acquisition from tiny point cloud images



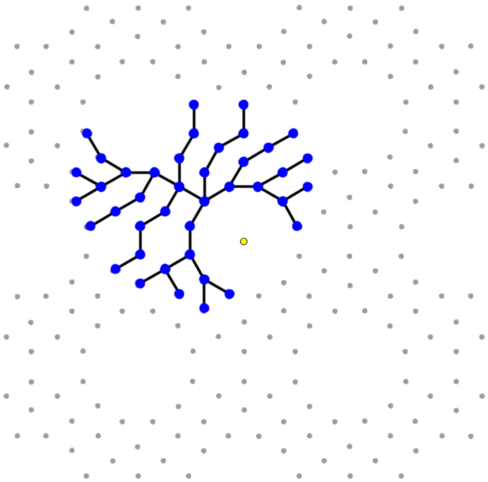
Automatic acquisition from tiny point cloud images



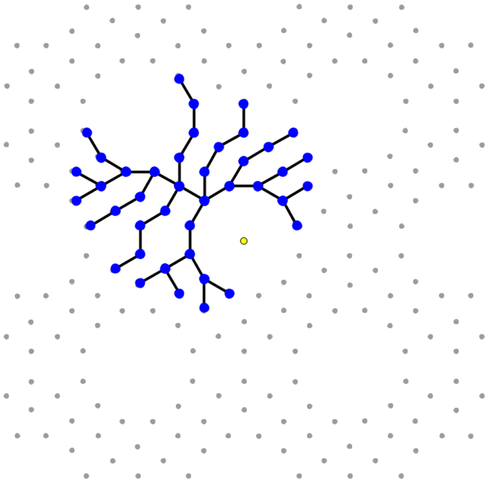
Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



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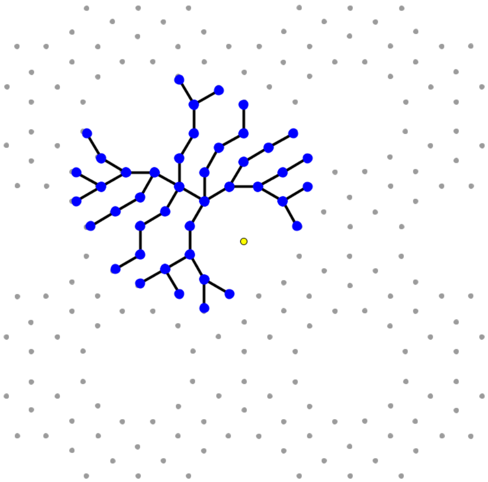
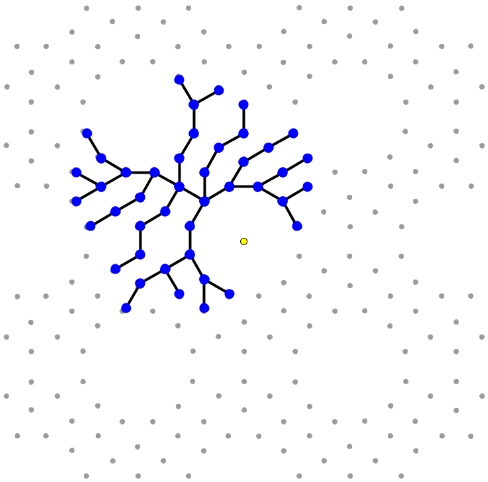


Automatic acquisition from tiny point cloud images



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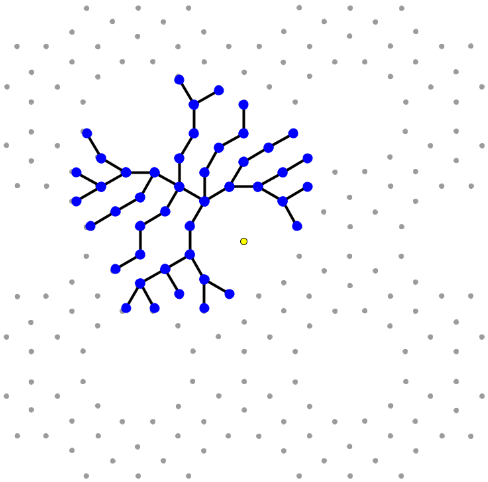

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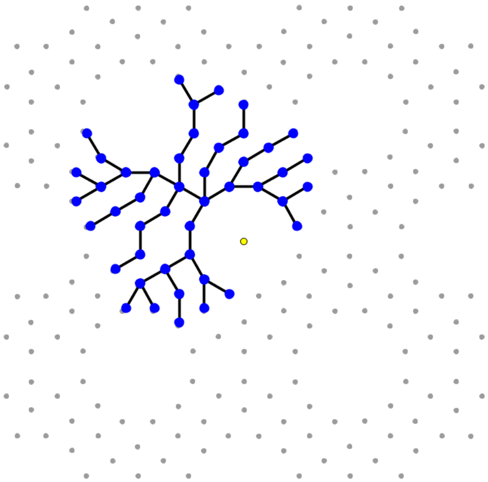

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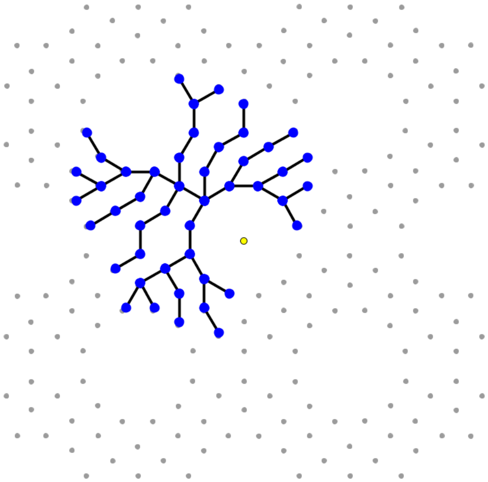

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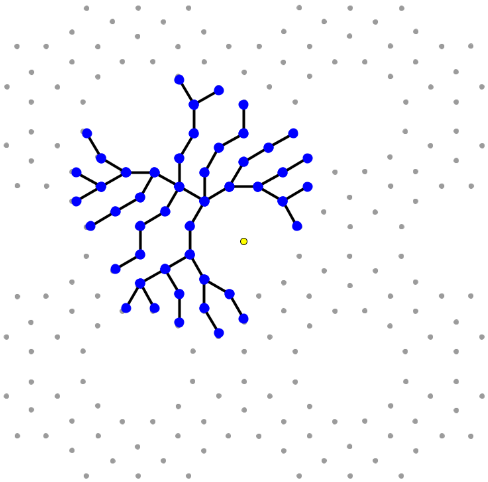
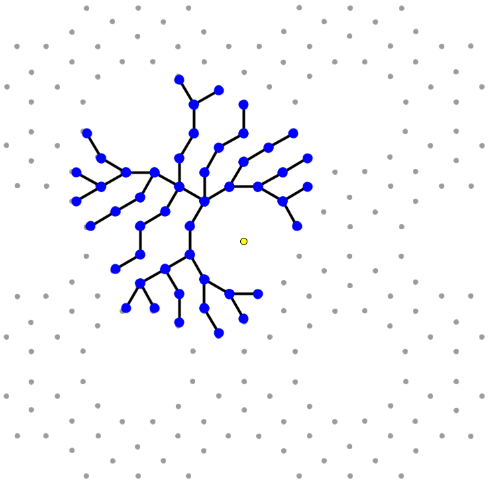
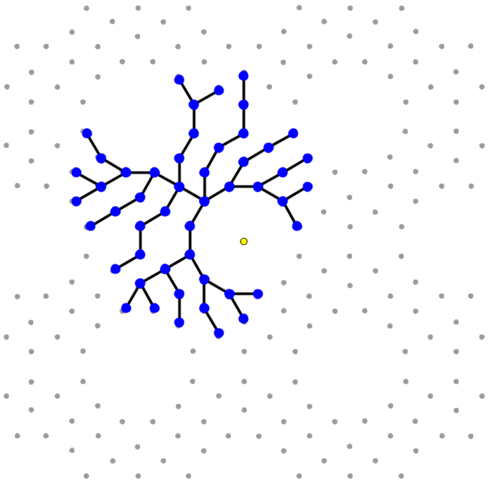


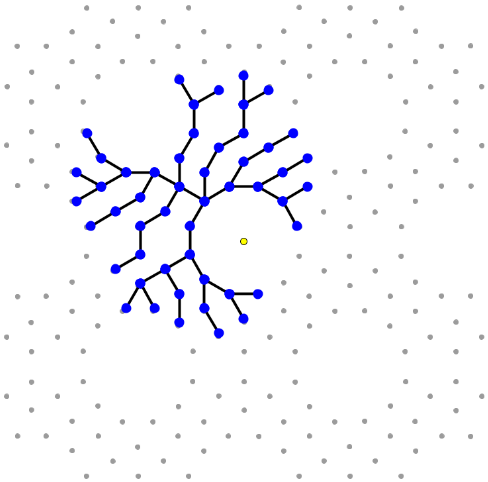
Automatic acquisition from tiny point cloud images



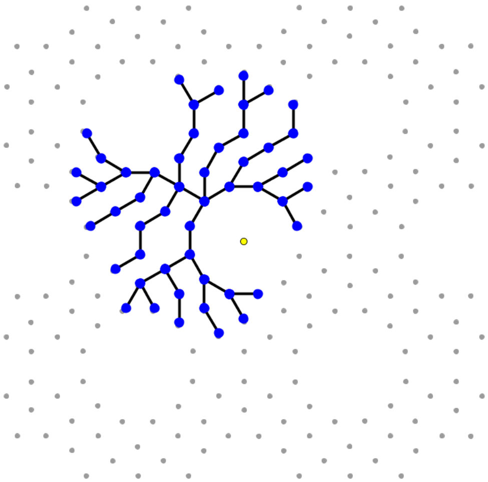
Automatic acquisition from tiny point cloud images



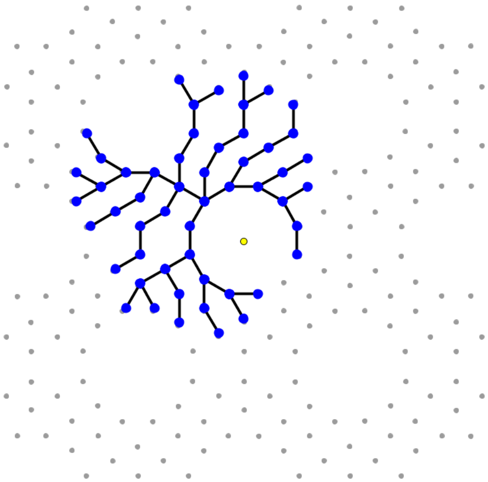
Automatic acquisition from tiny point cloud images



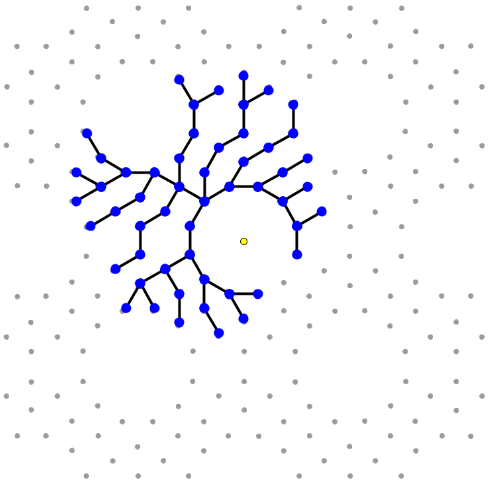
Automatic acquisition from tiny point cloud images



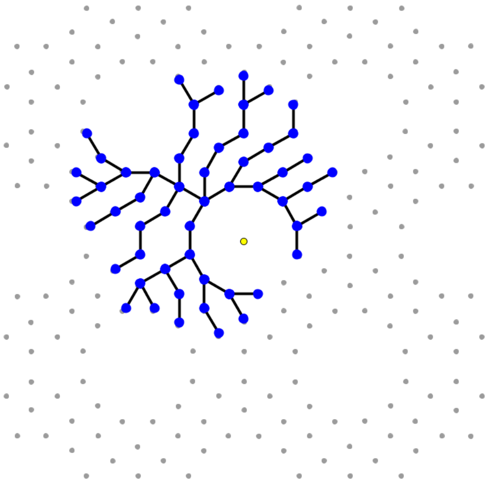
Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



Automatic acquisition from tiny point cloud images



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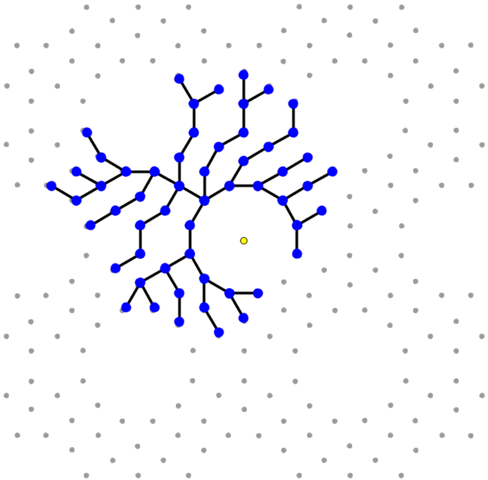

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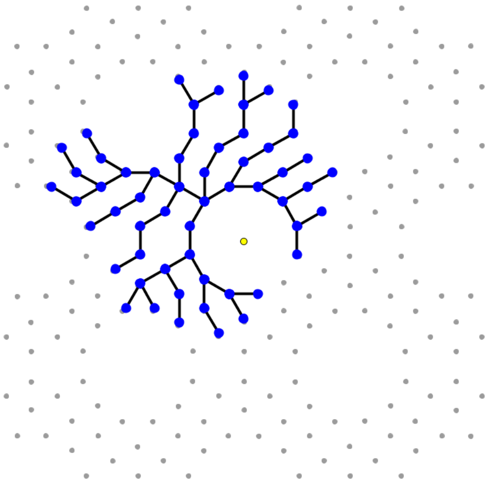

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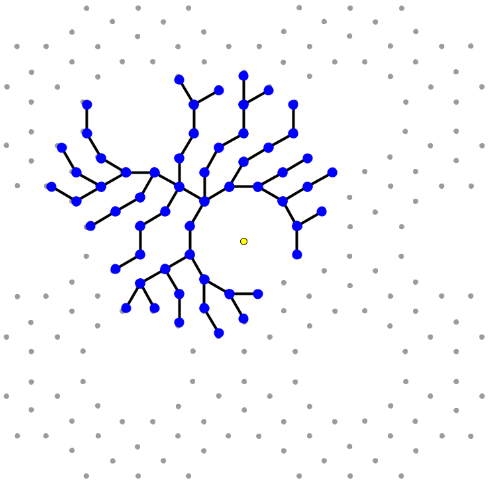

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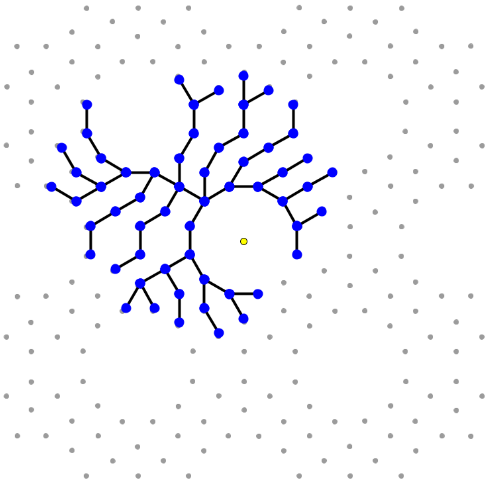

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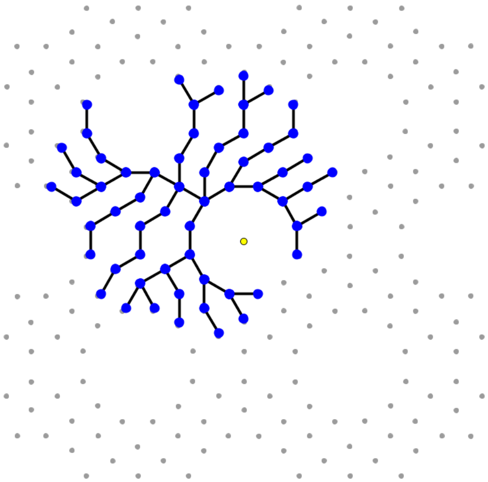

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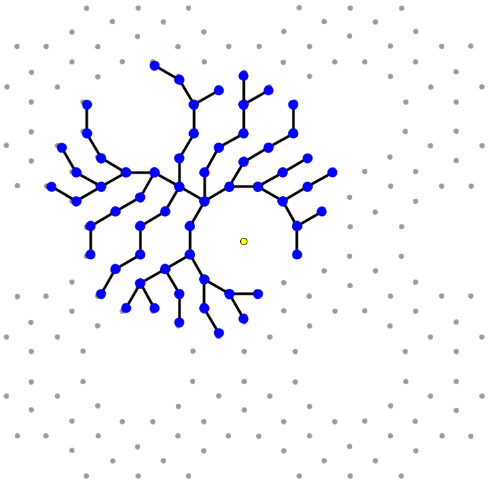

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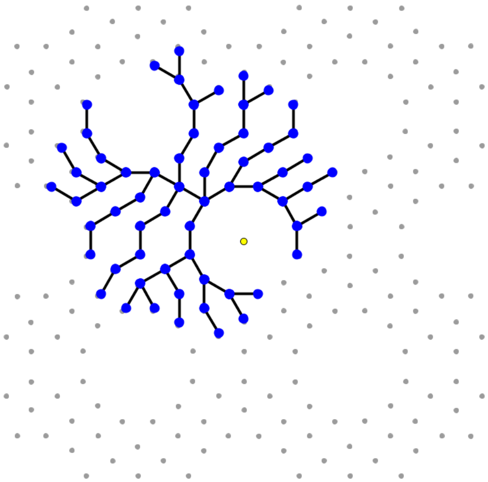

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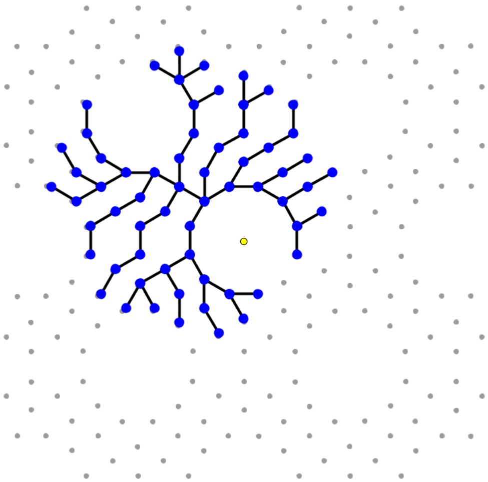

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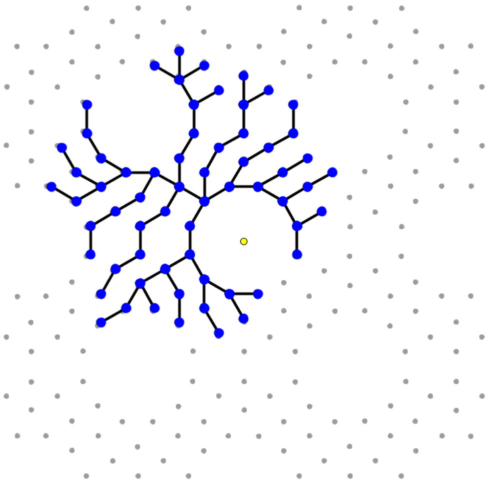

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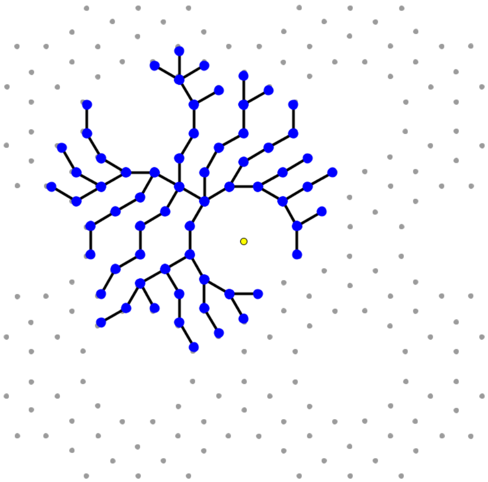

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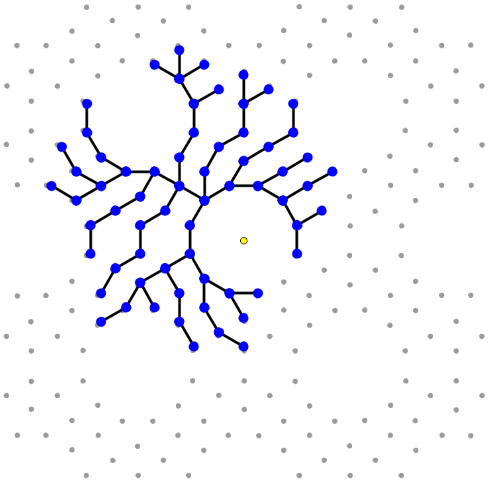

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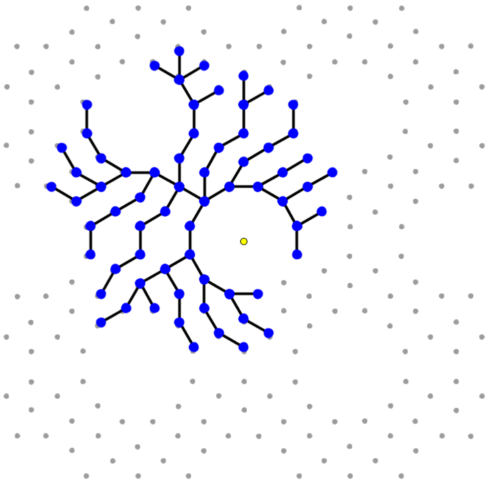

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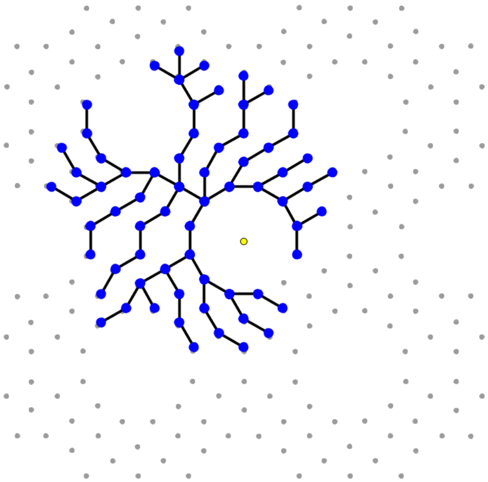

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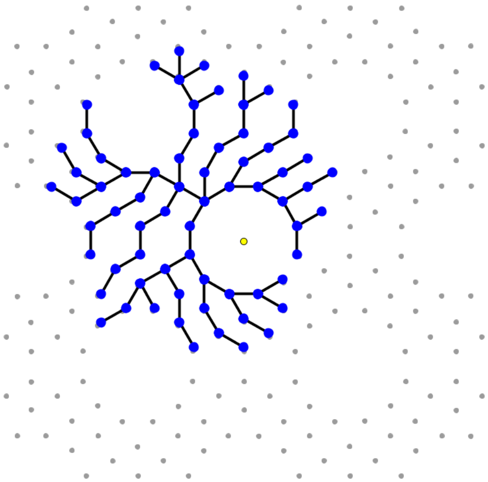

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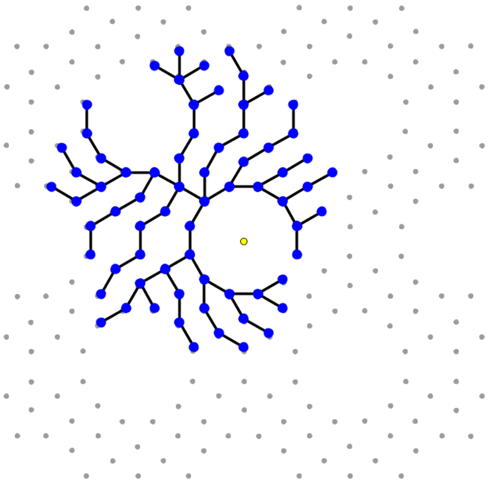

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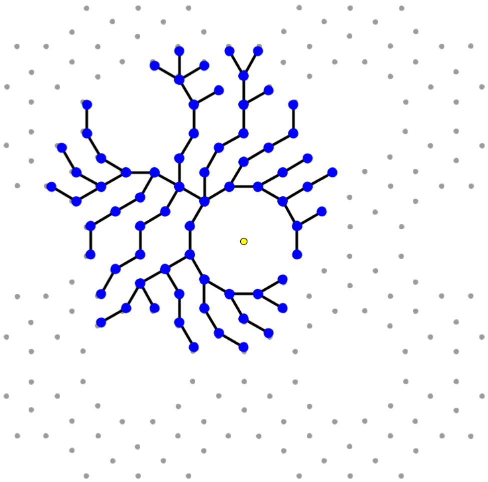

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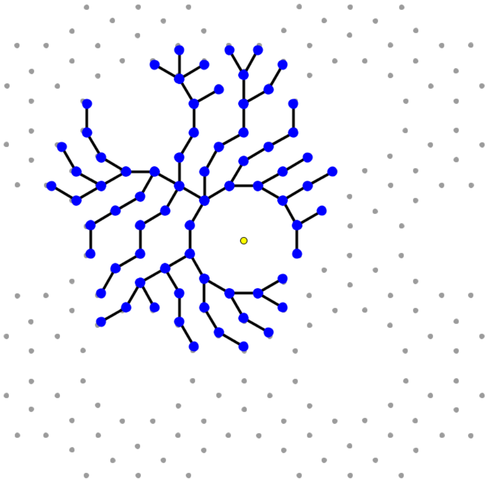

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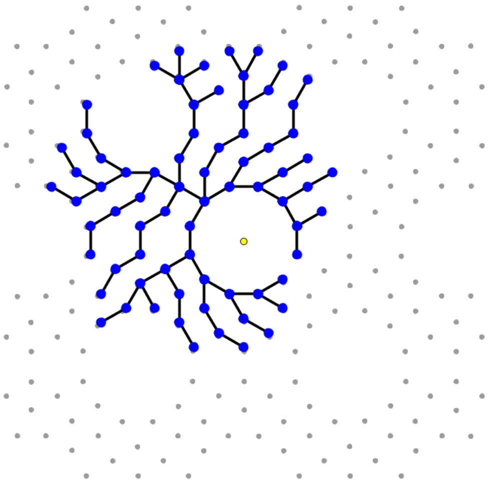

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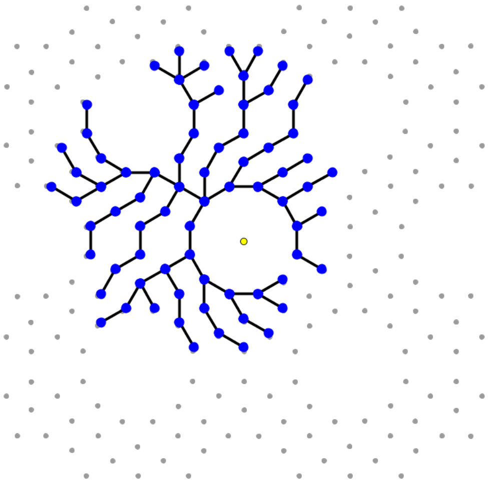

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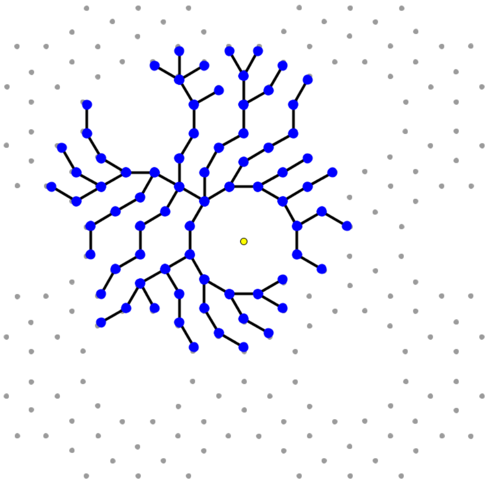

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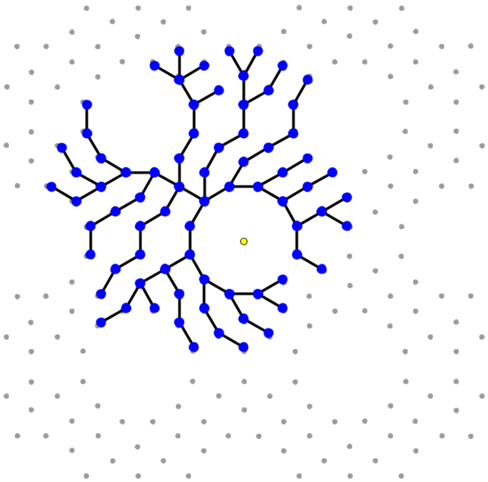

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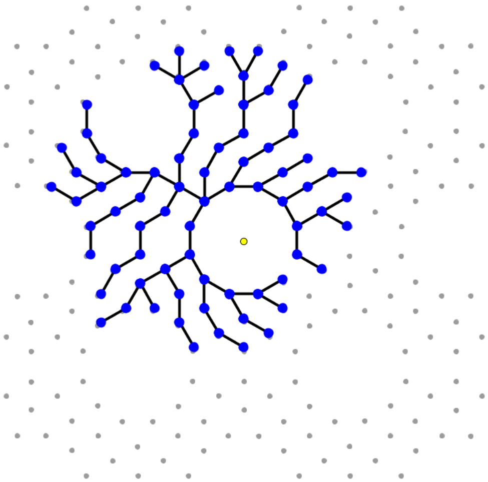

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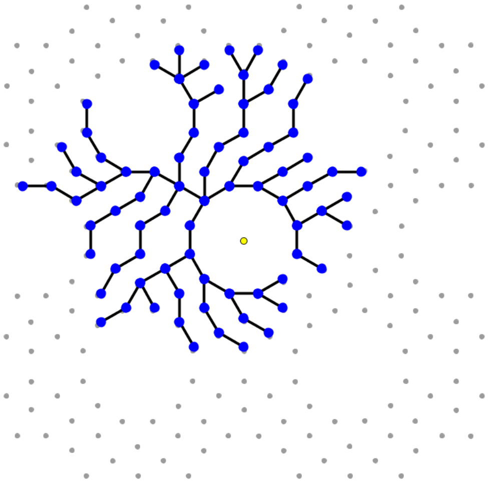

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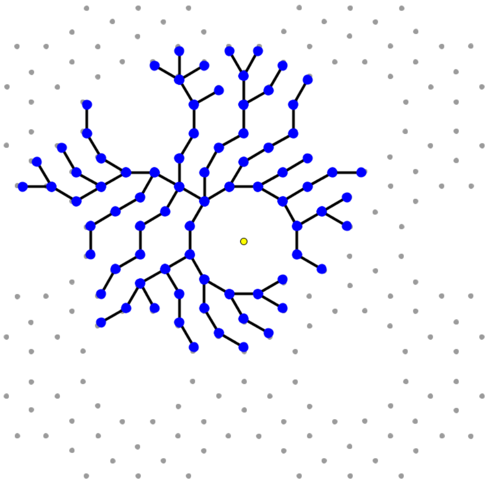

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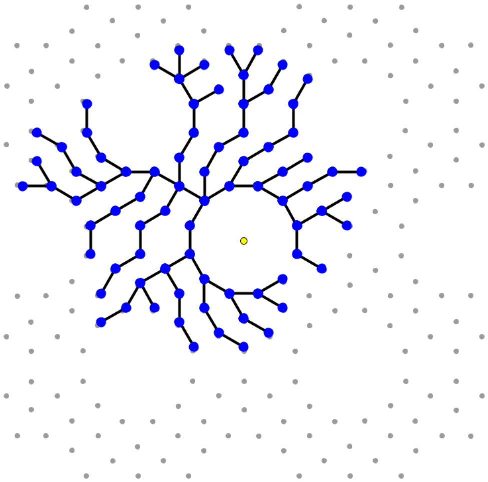

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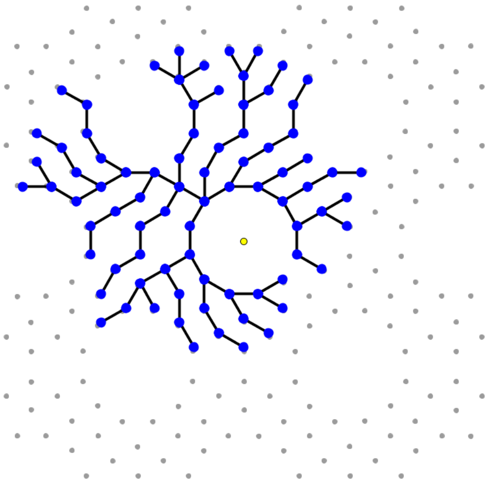

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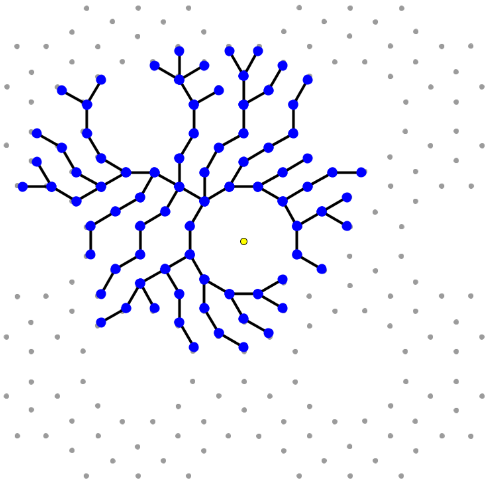

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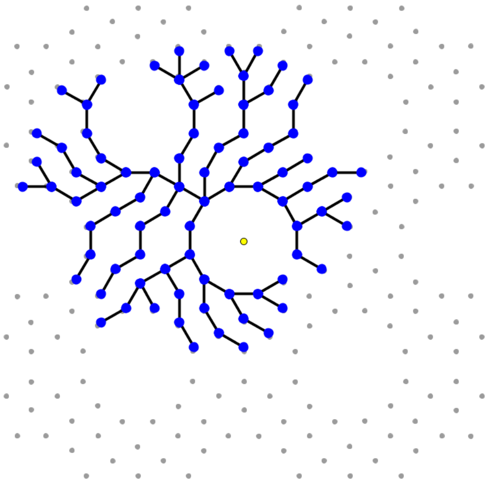

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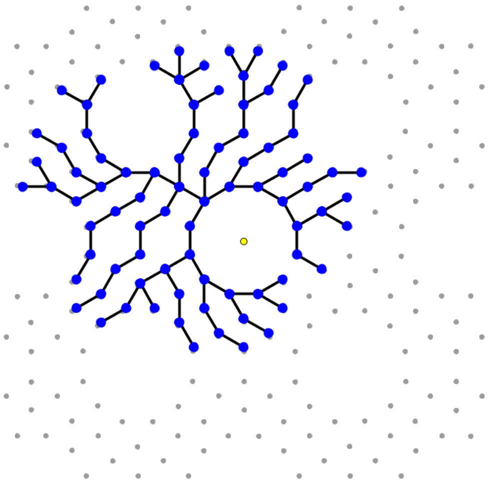

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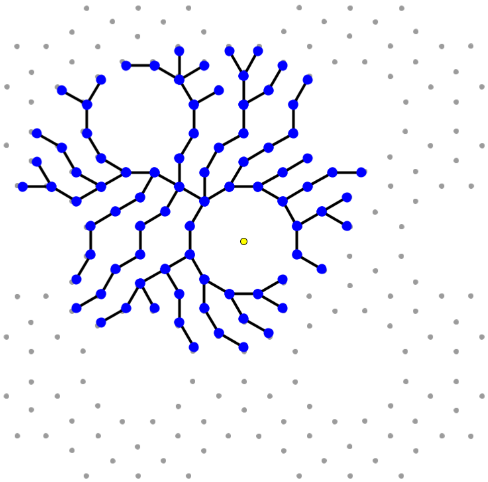

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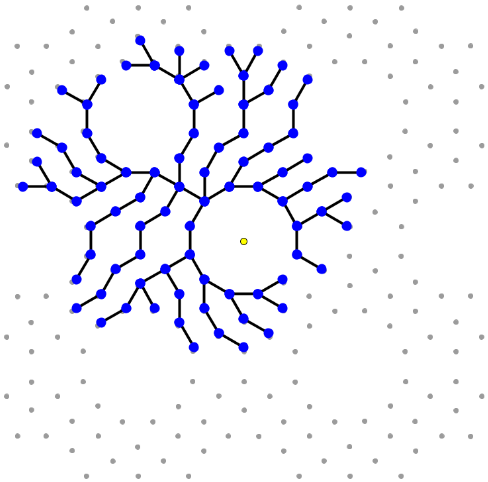

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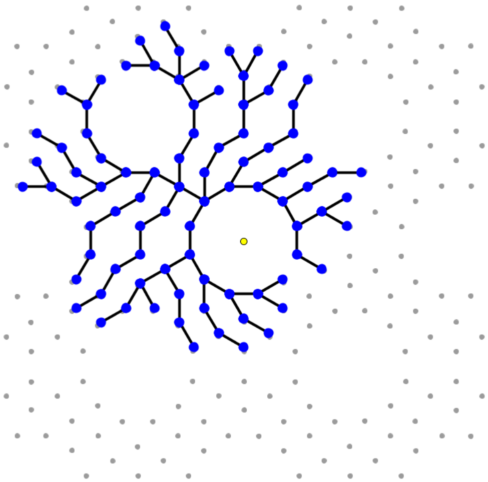

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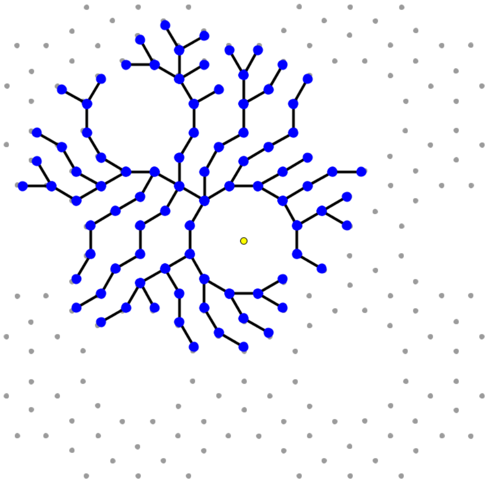

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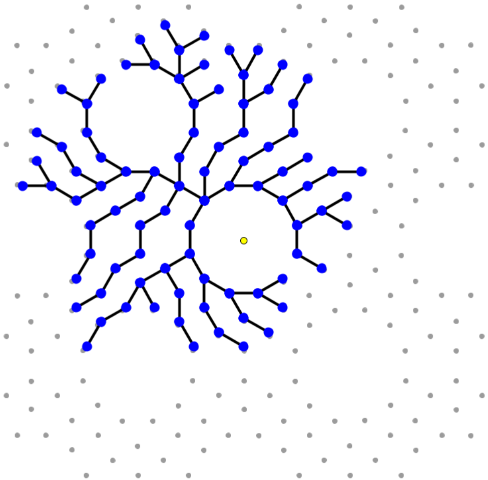

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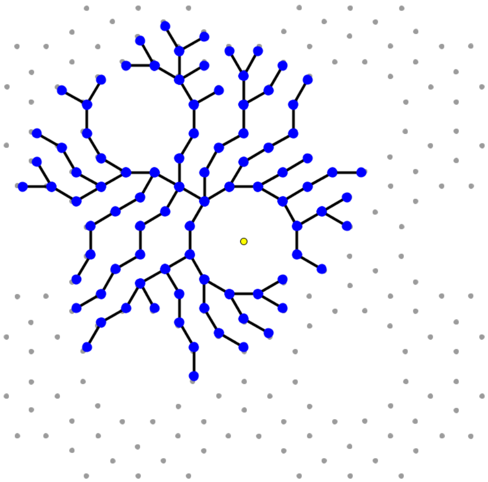

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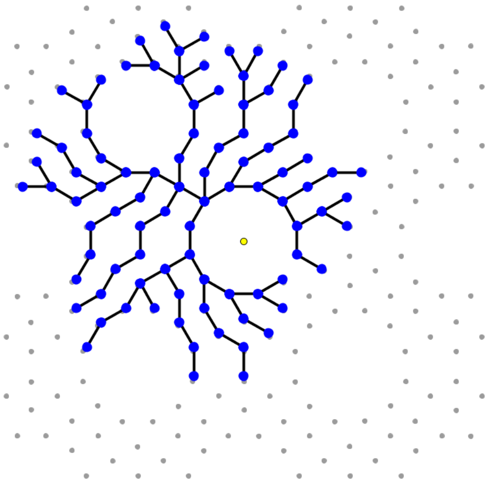

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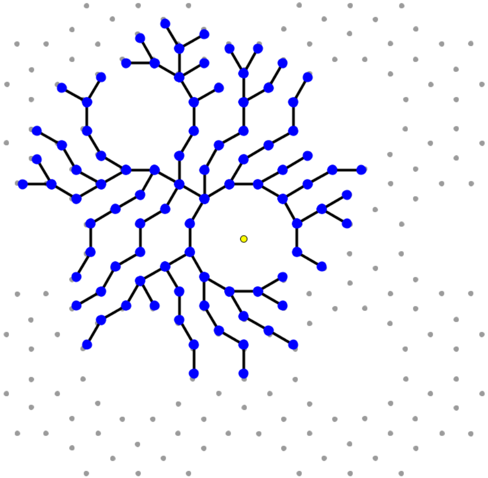

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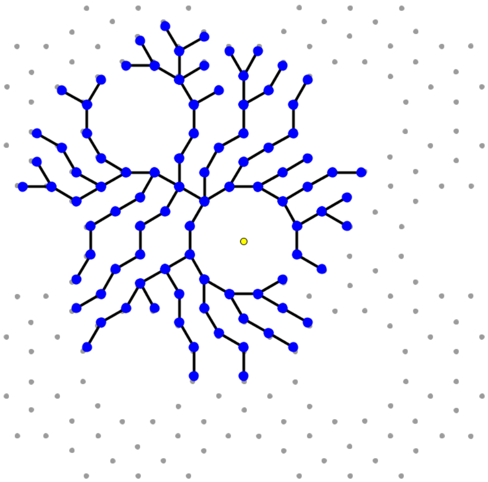

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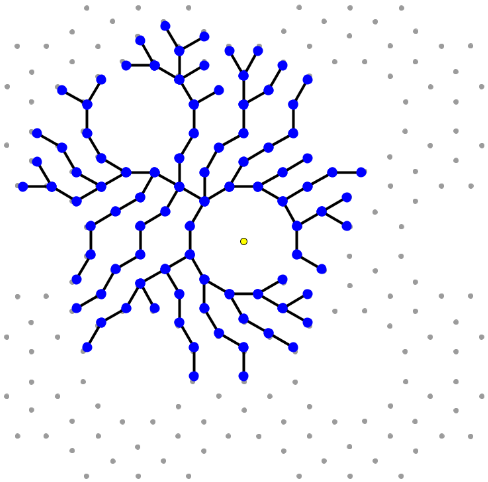

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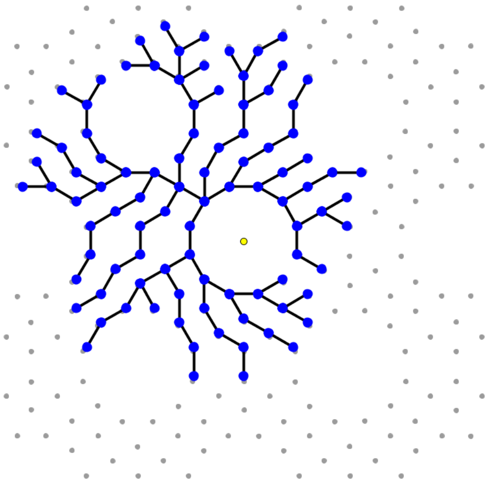

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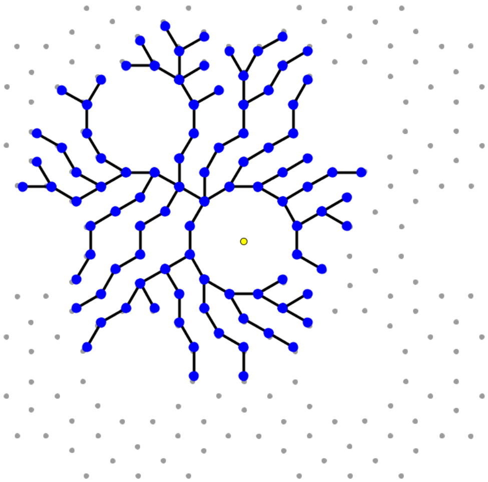

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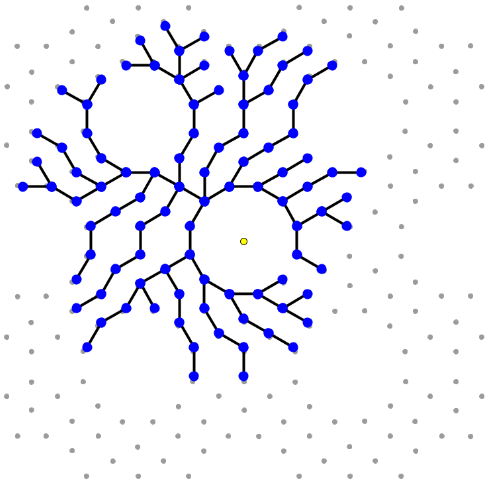

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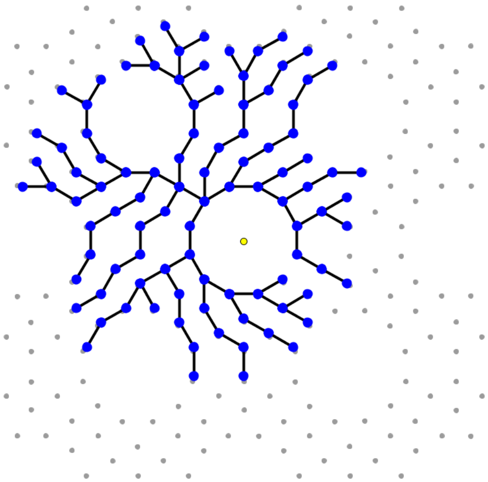

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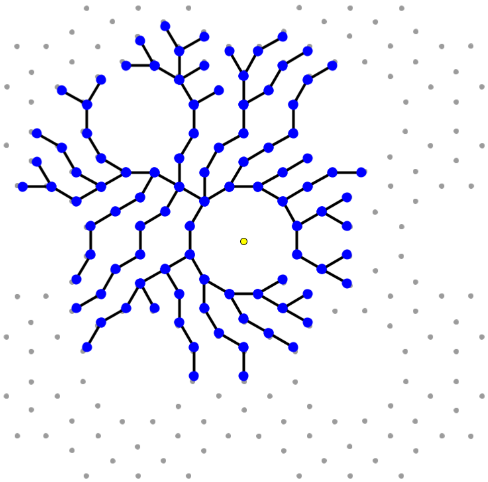

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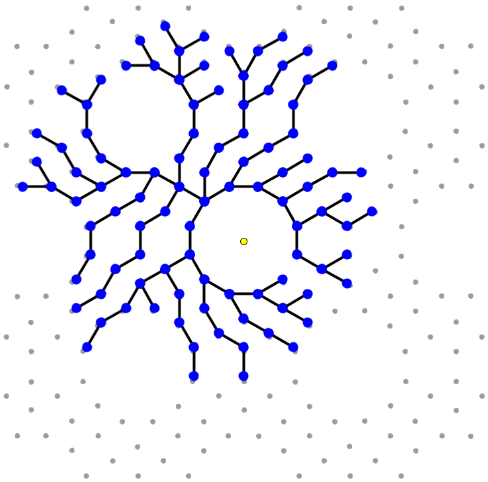

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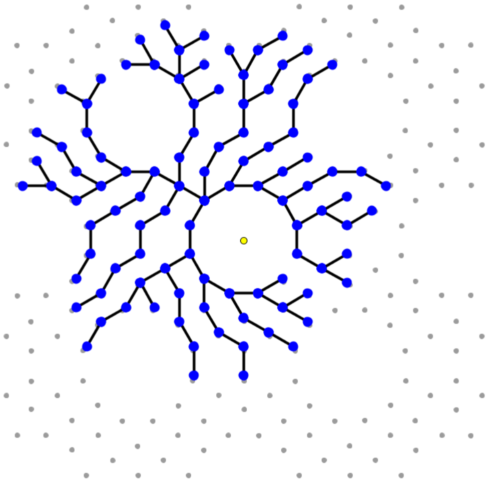

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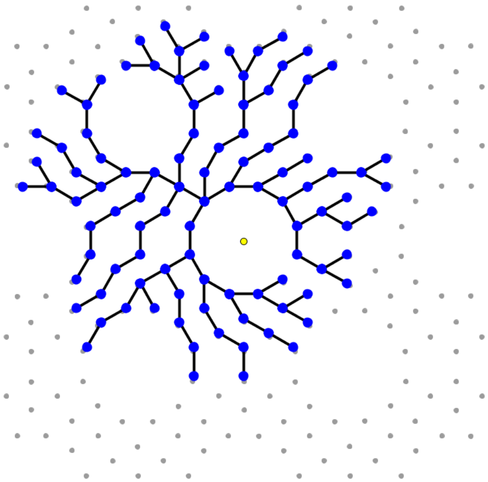

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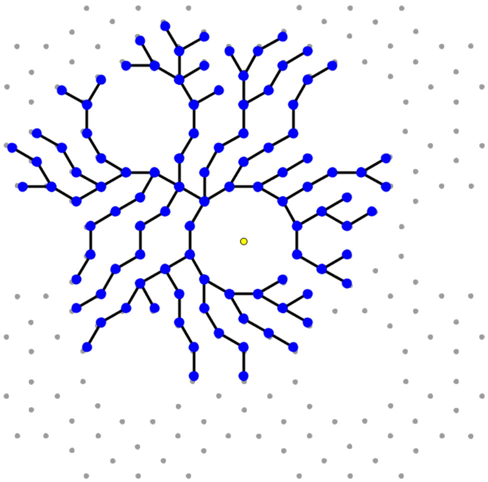

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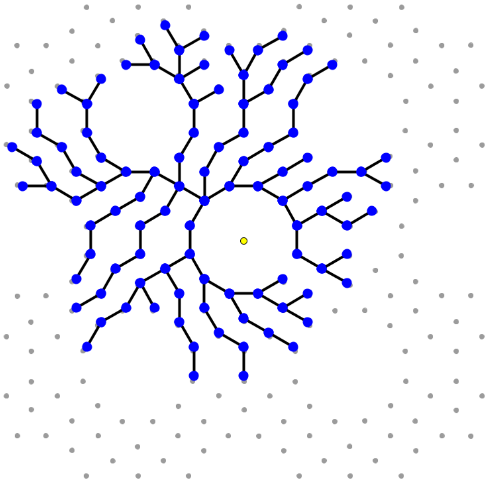

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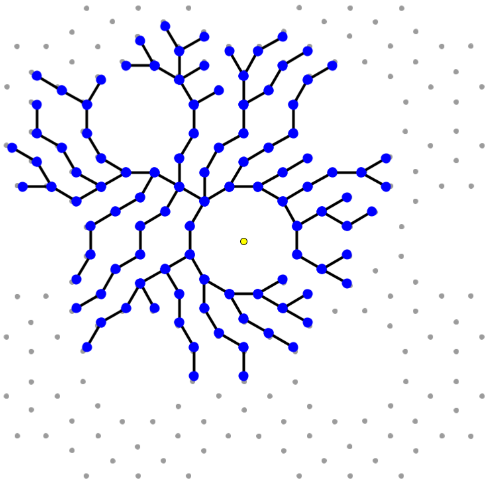

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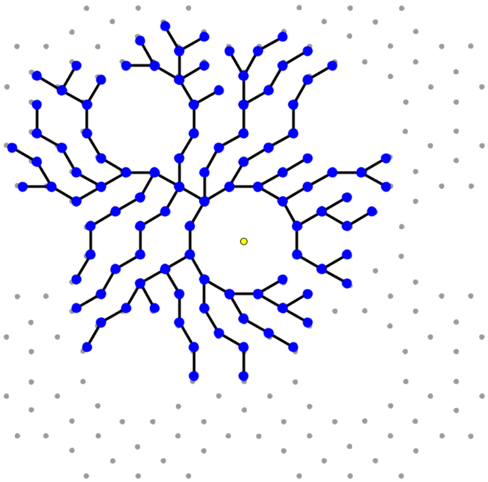

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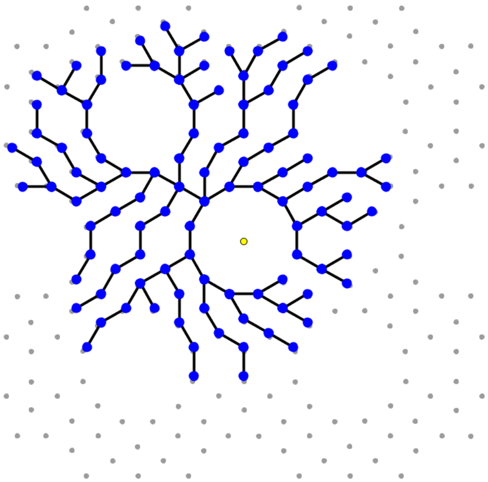

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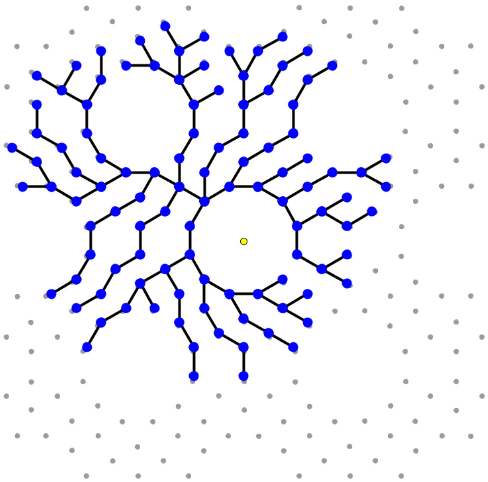

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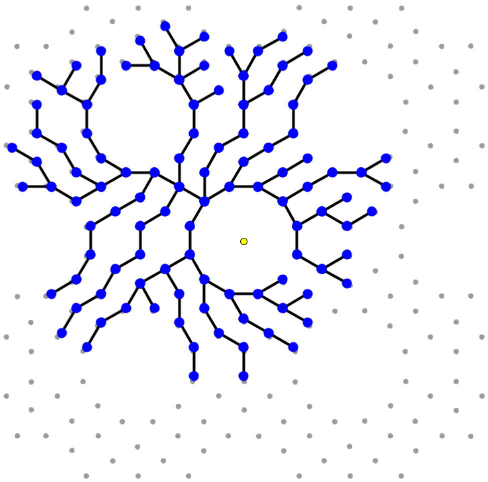

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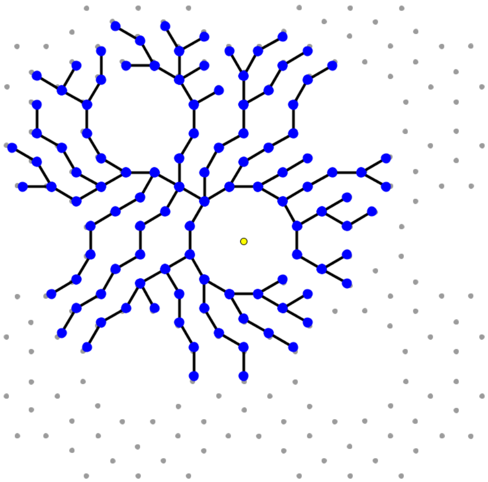

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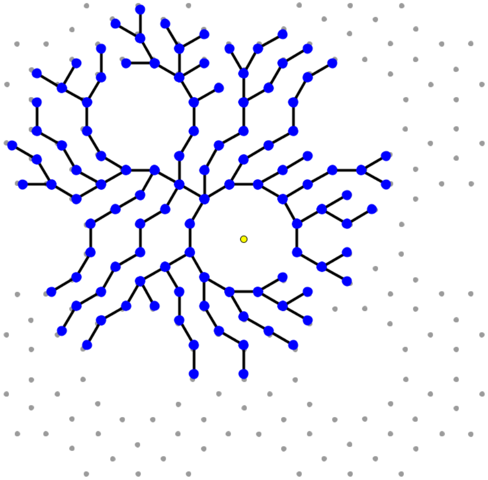

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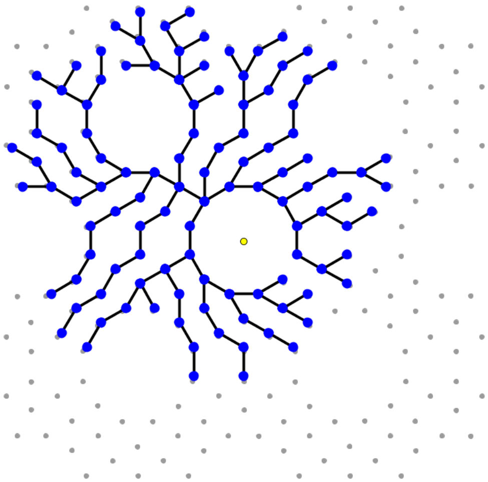

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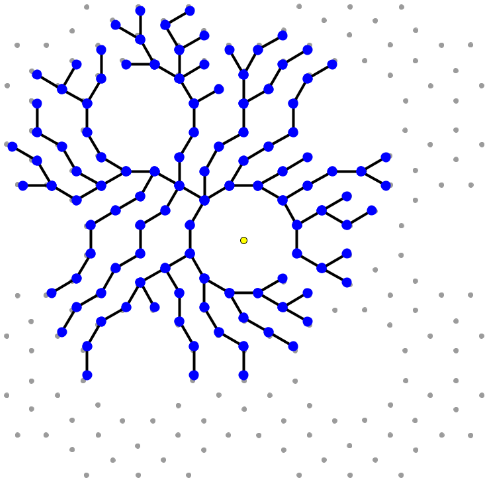

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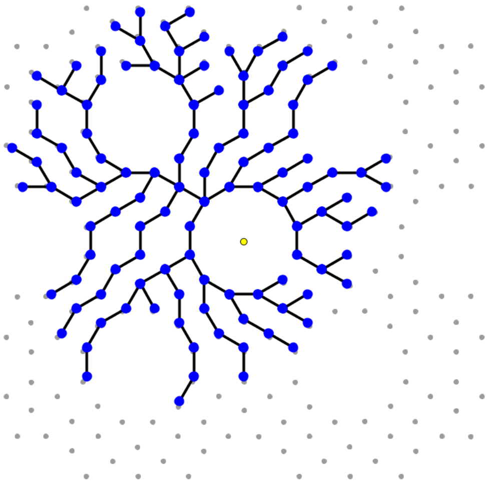

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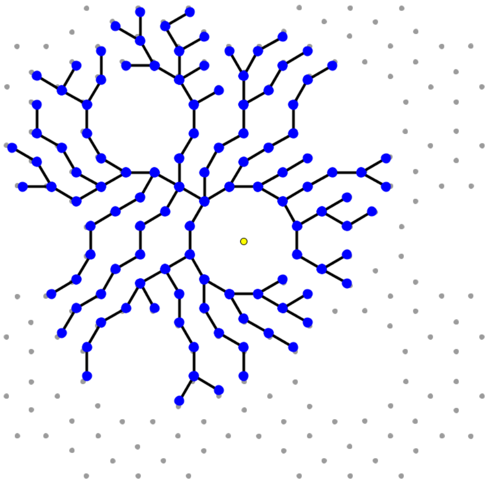

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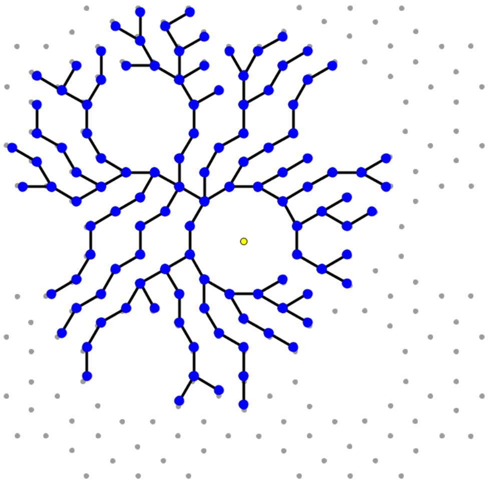

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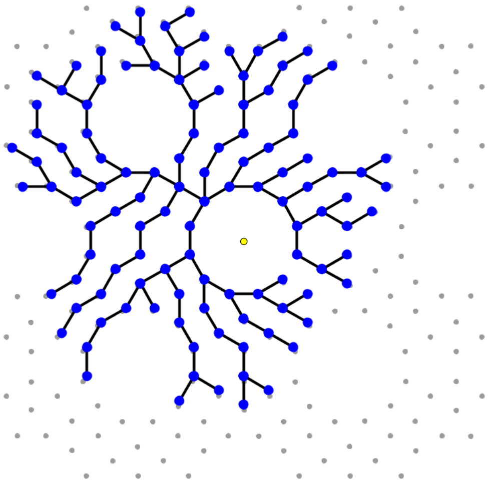

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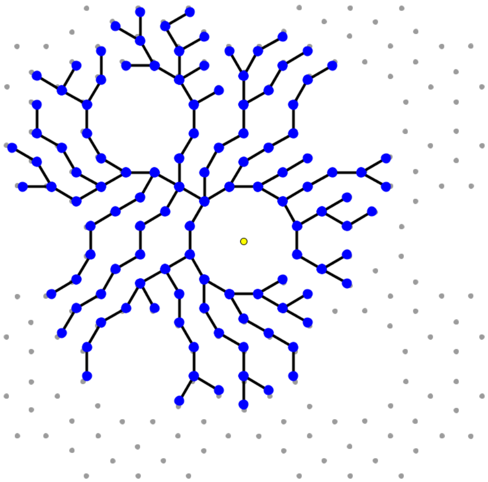

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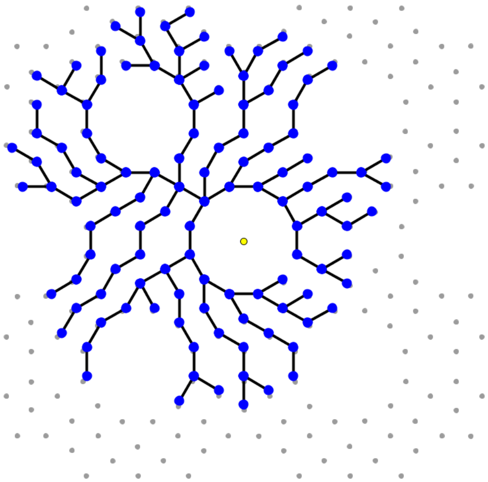

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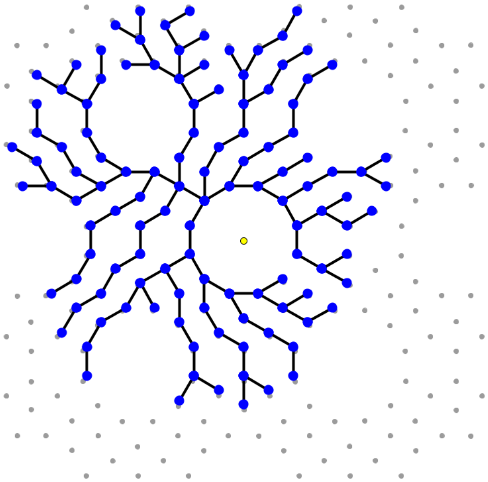

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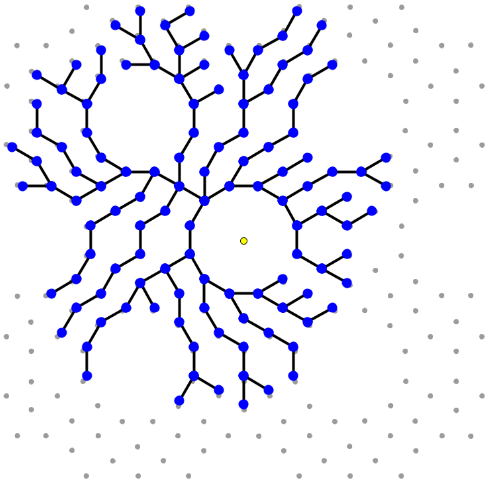

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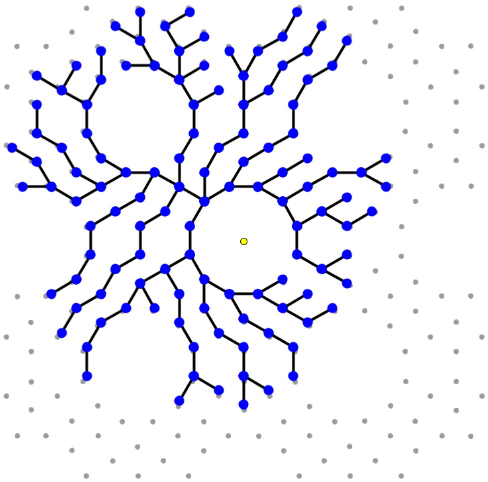

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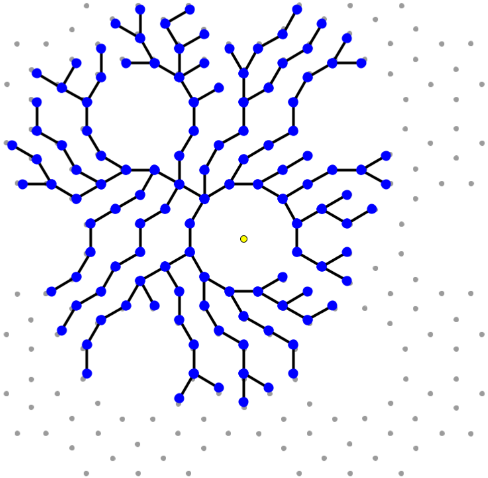

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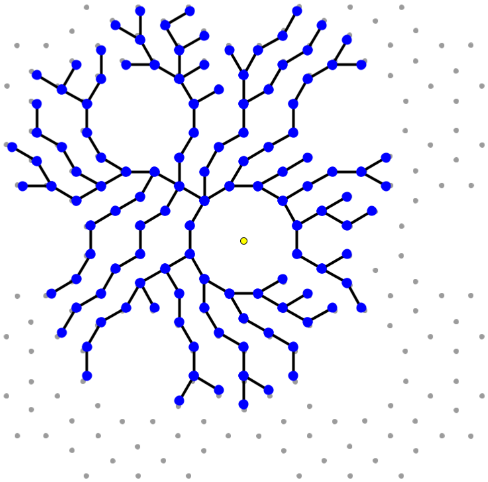

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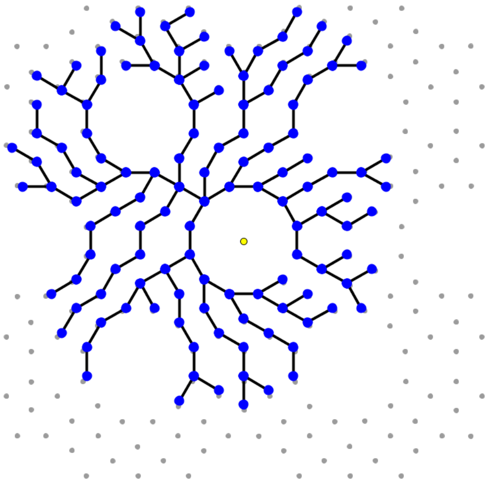

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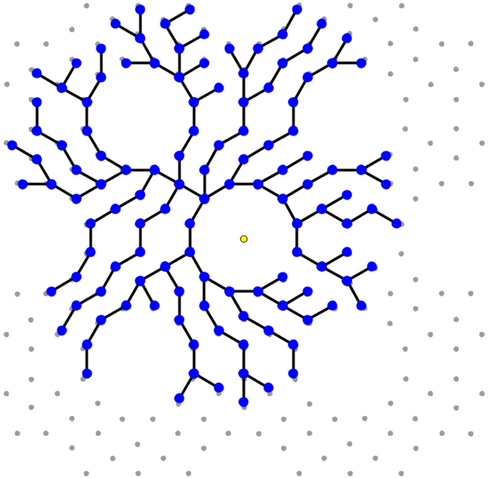

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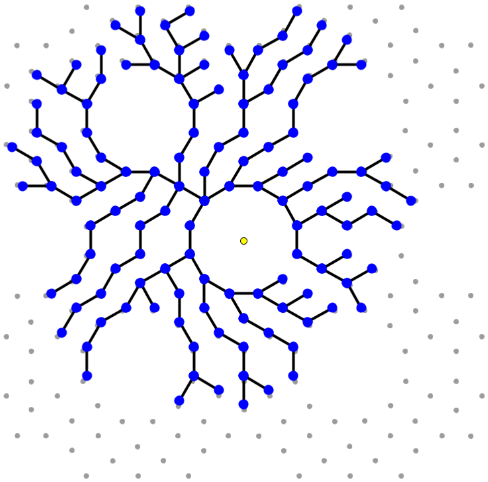

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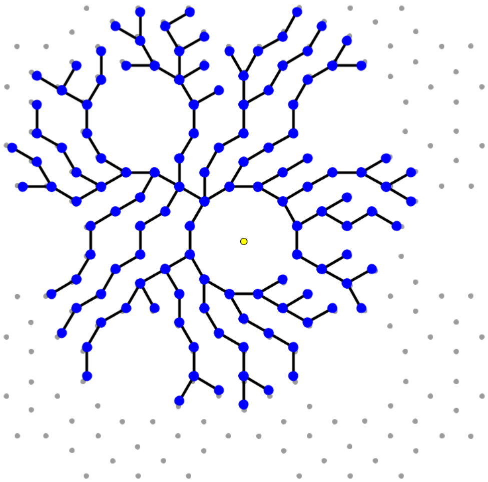

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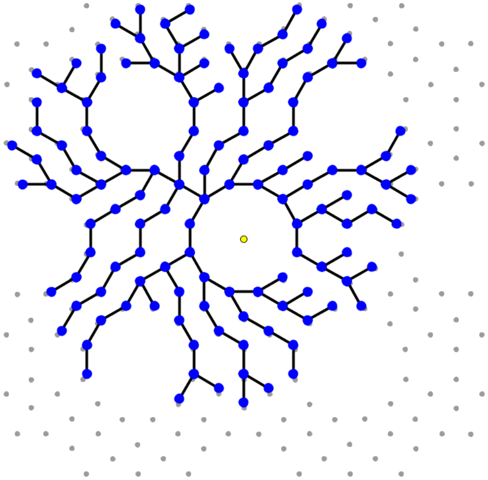

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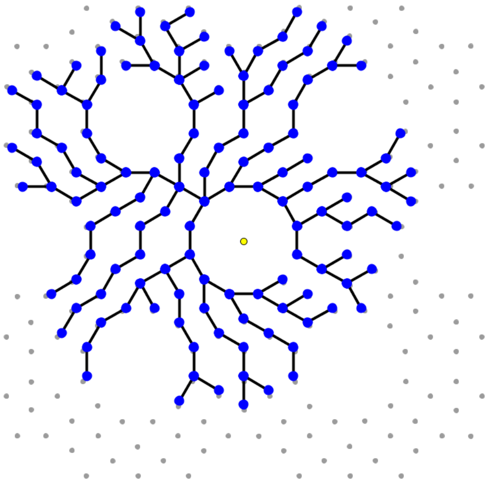

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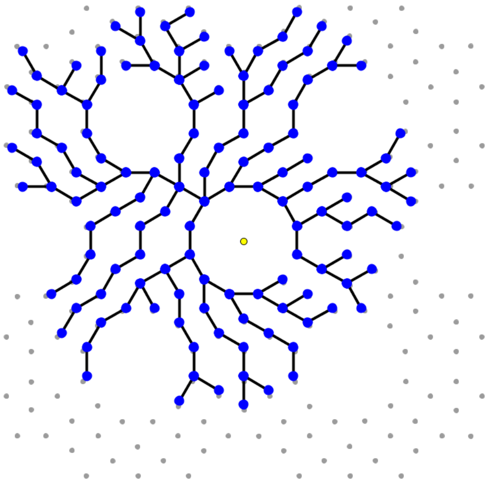

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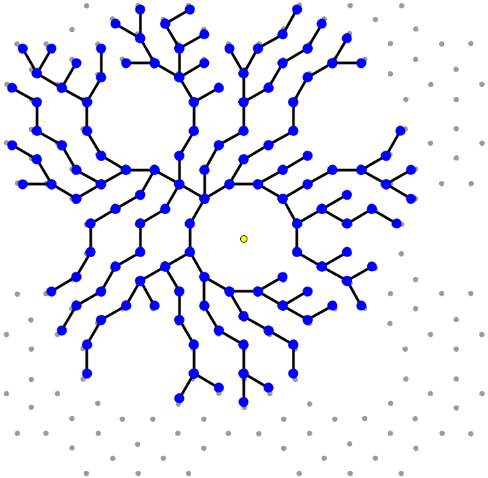

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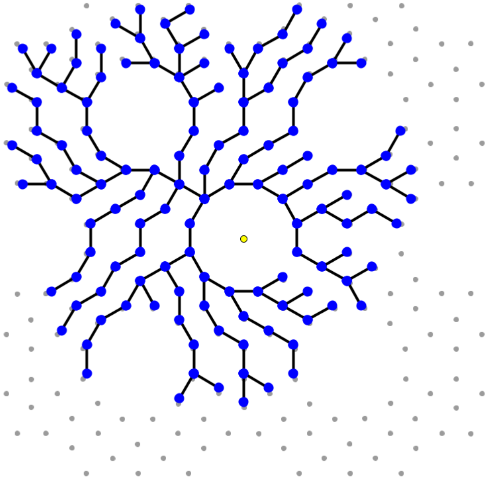

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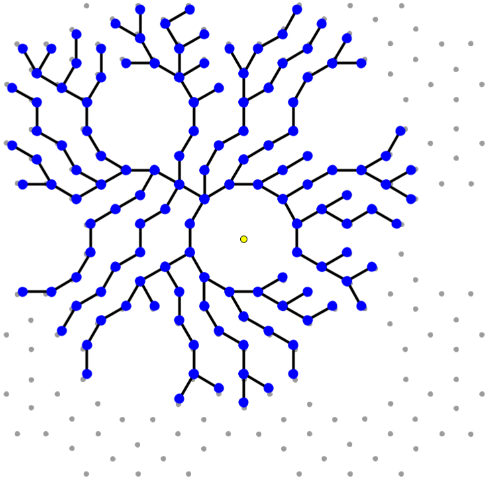

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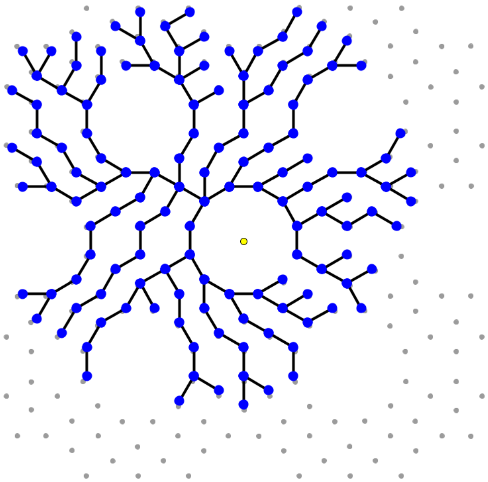

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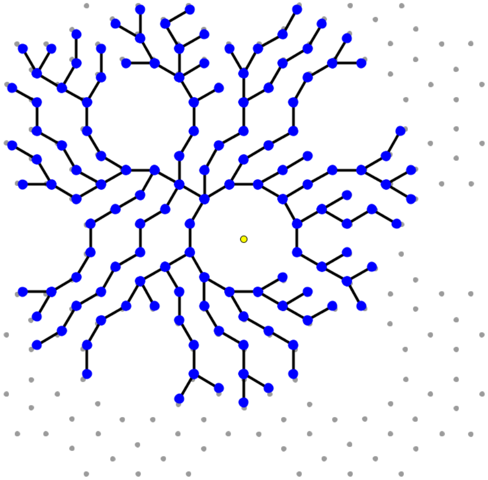

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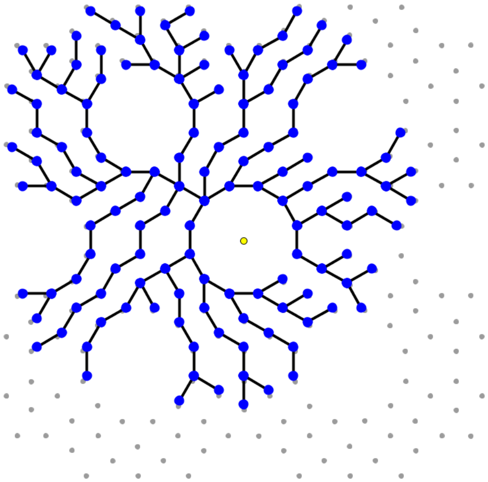

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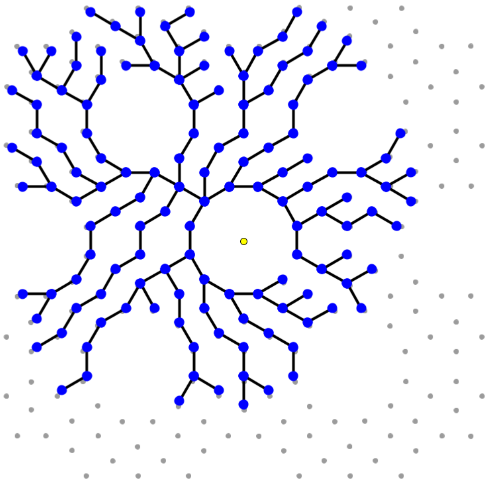

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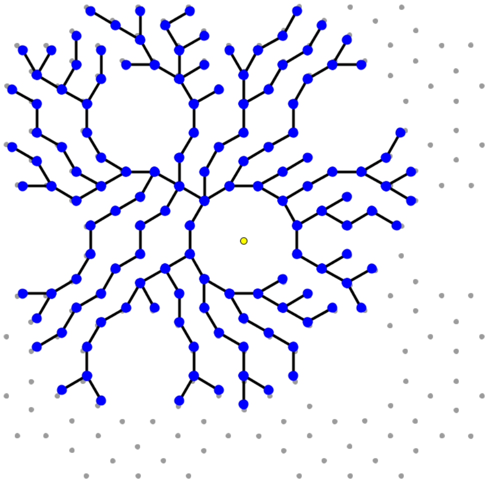

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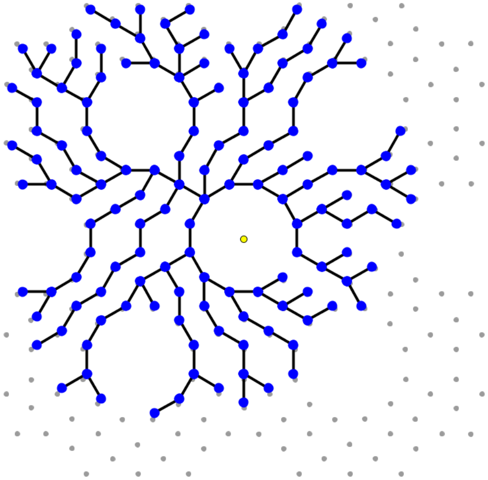

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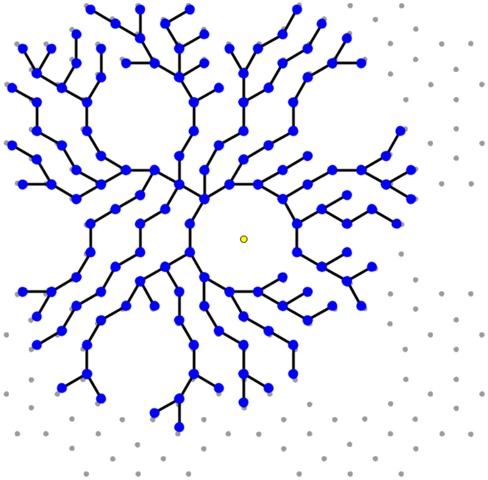

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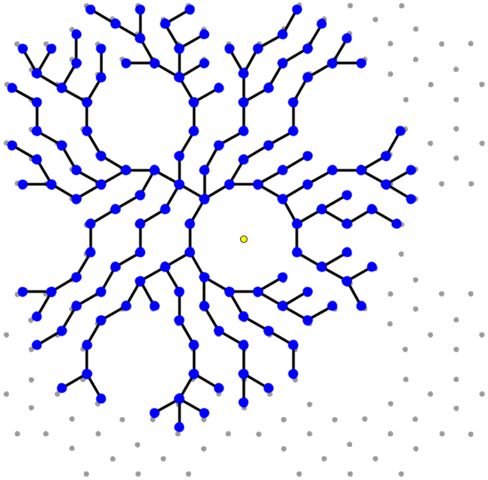

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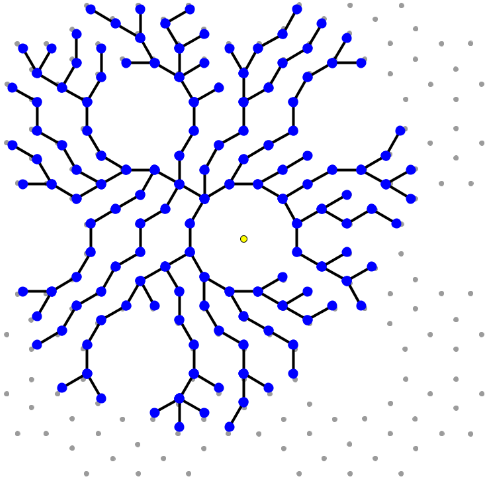

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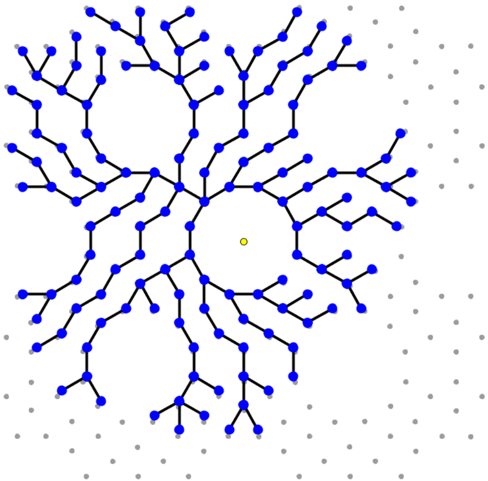

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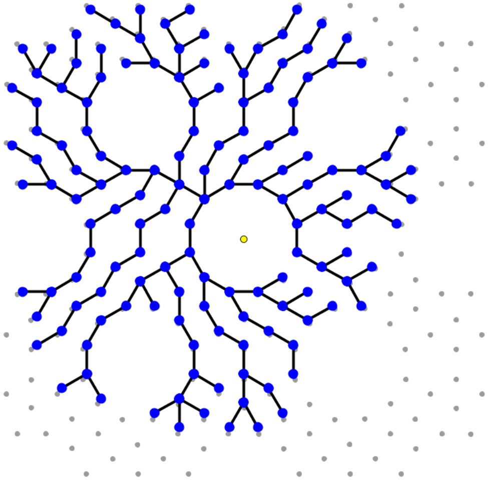

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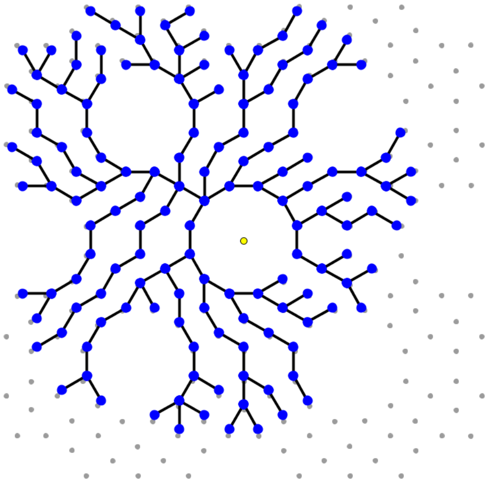

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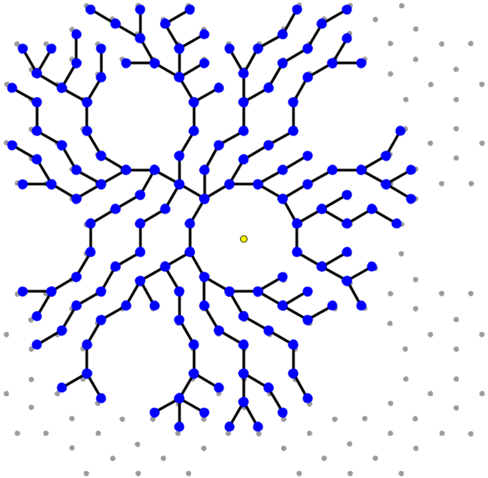

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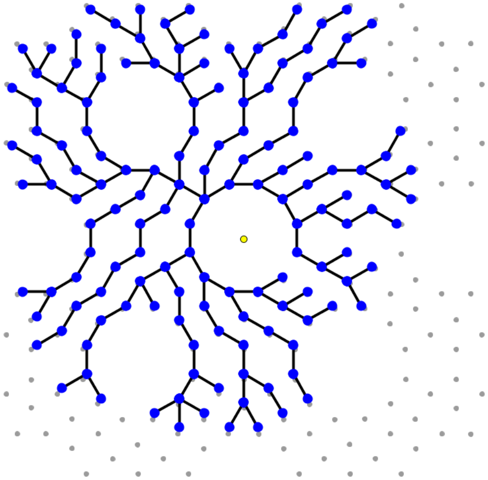

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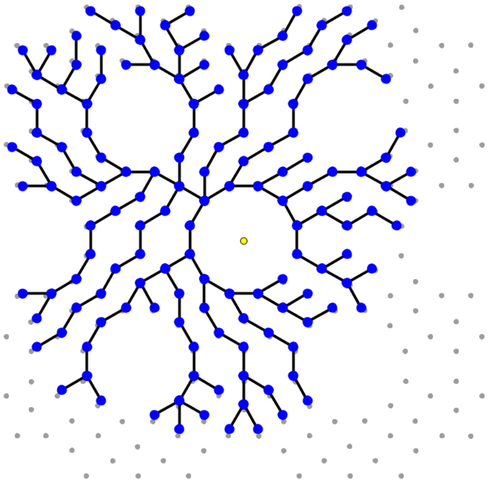

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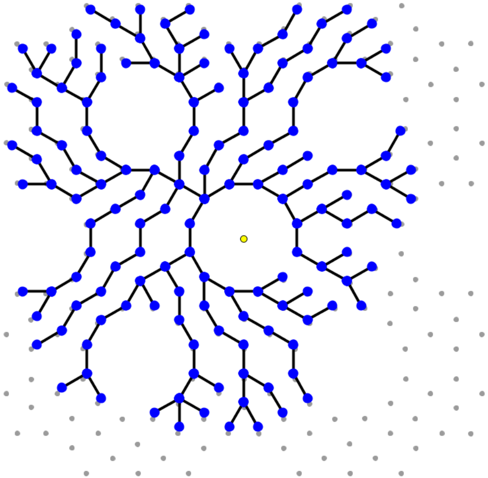

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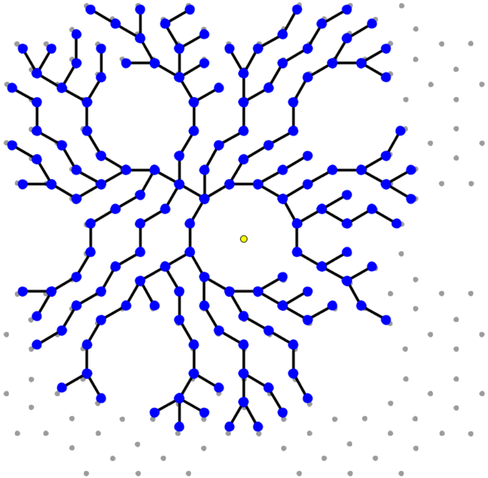

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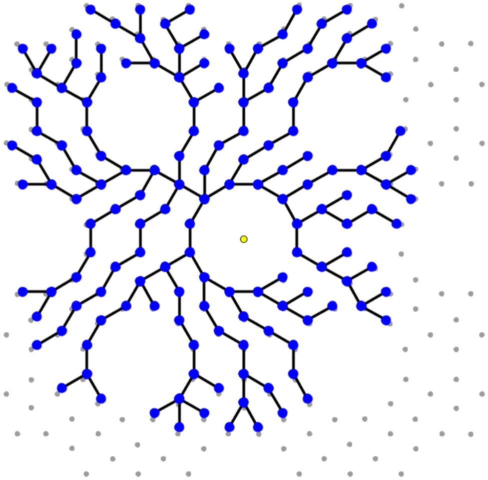

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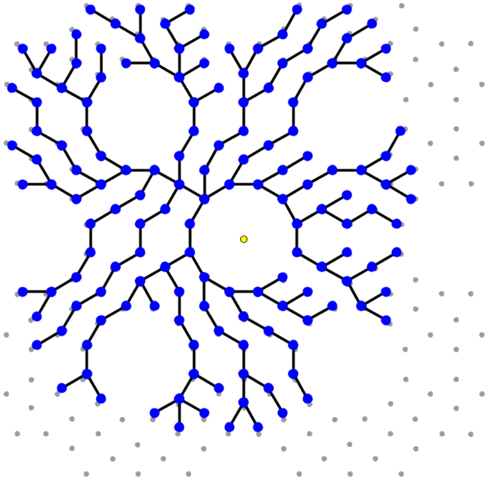

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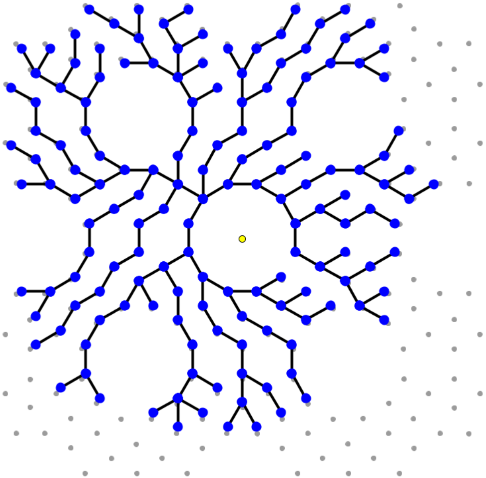

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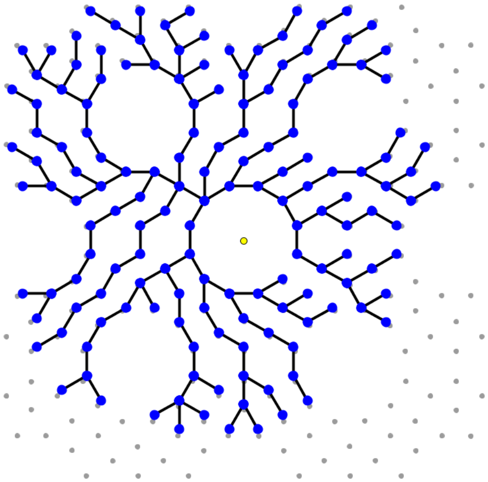

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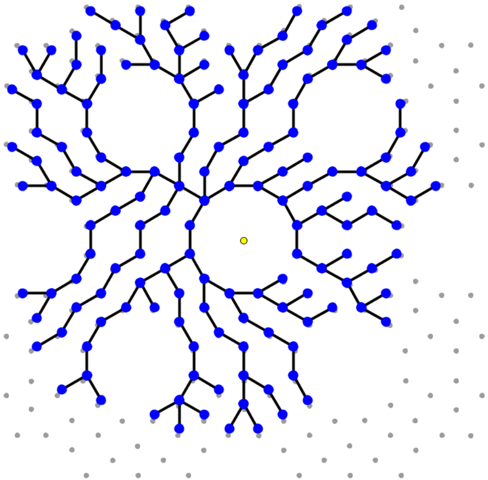

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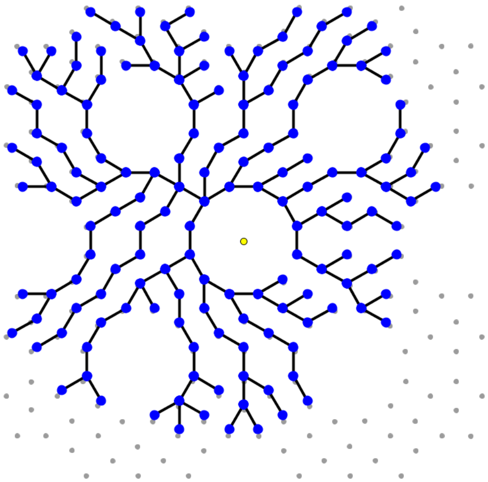

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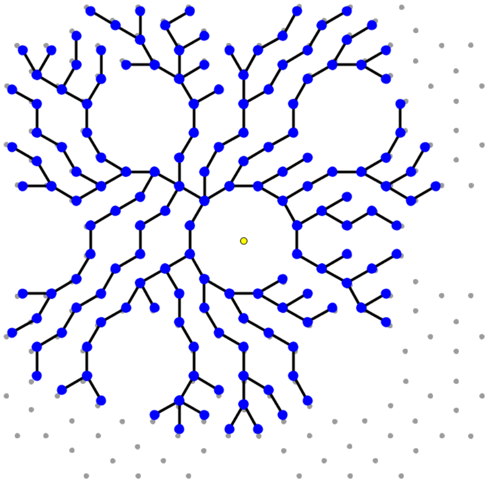

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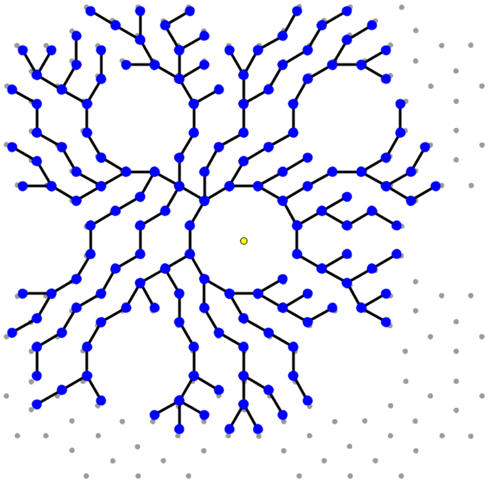

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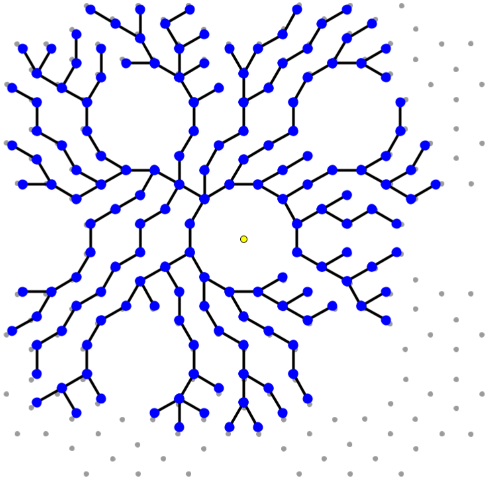

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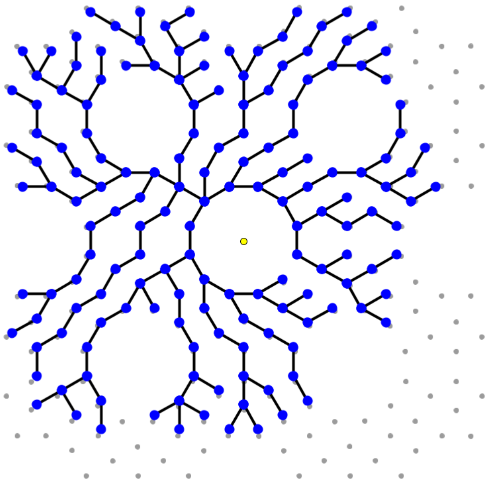

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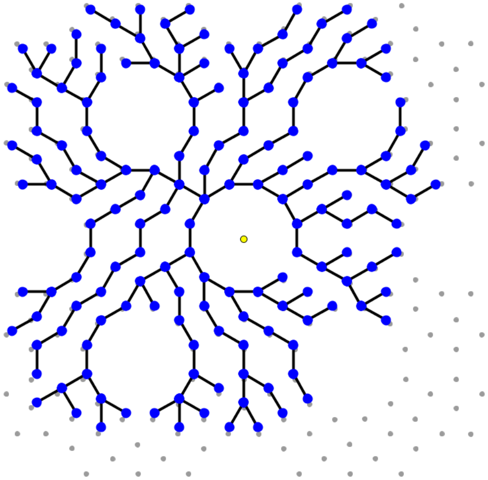

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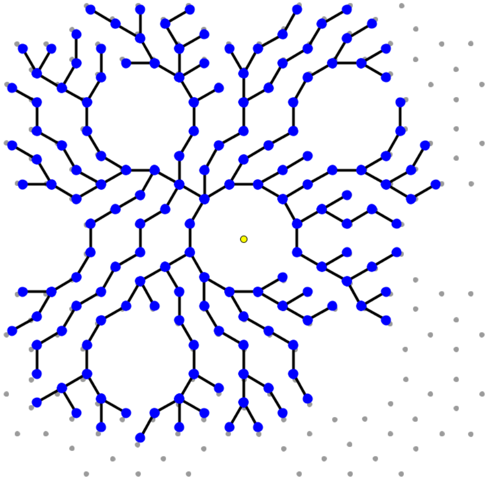

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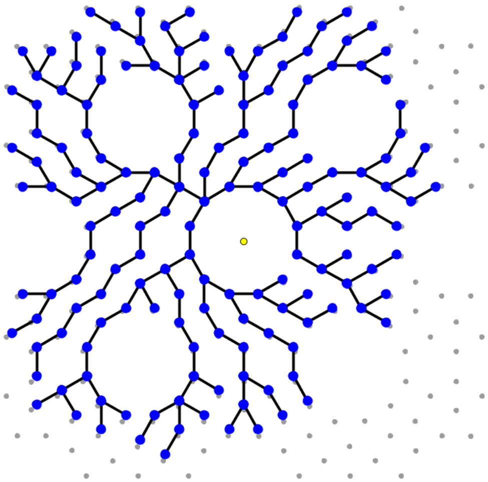

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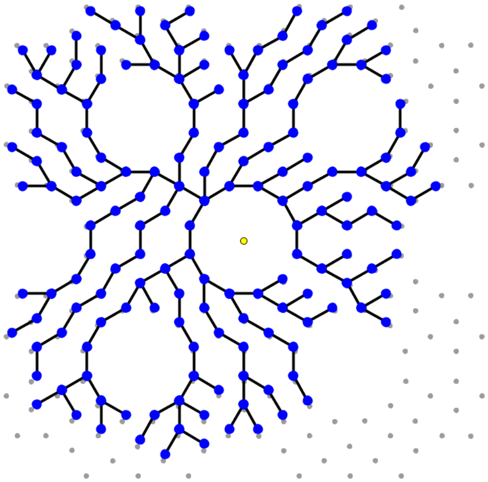

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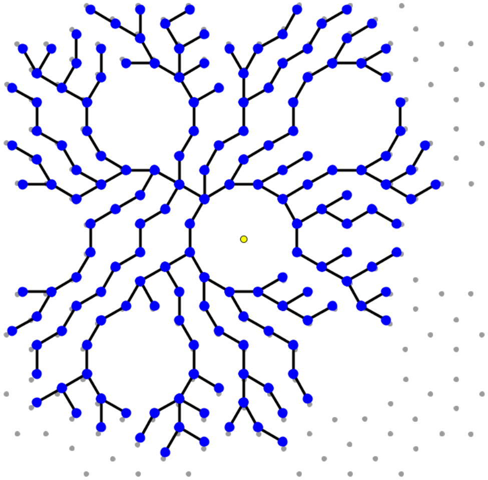

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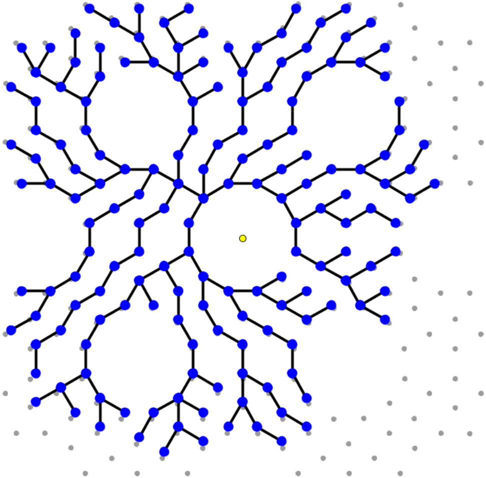

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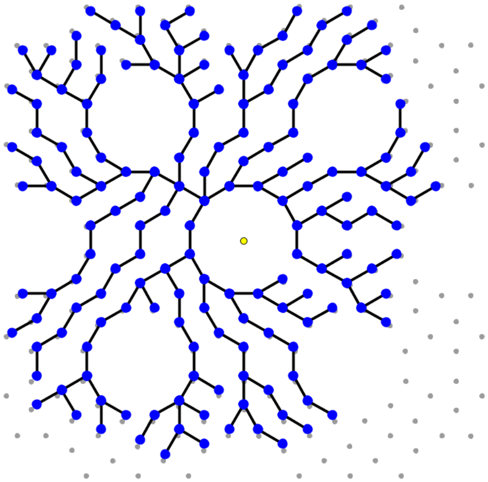

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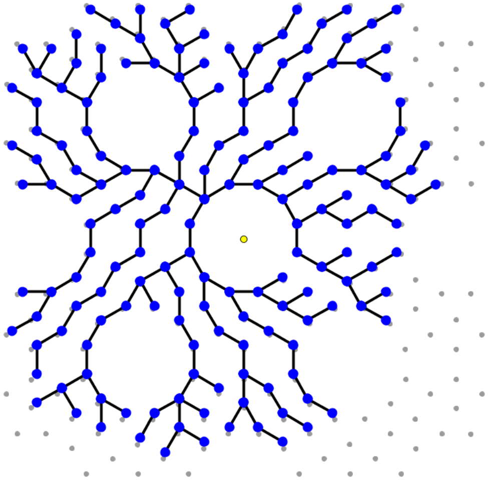

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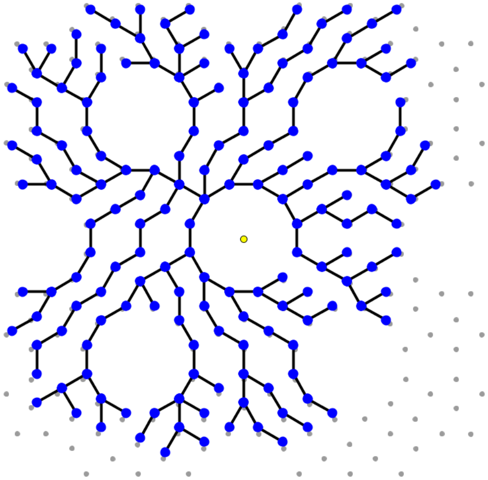

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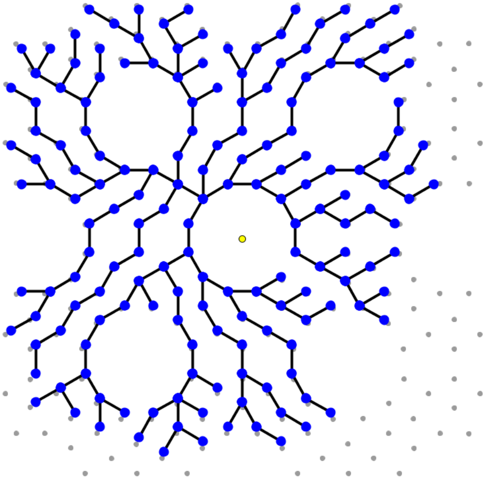

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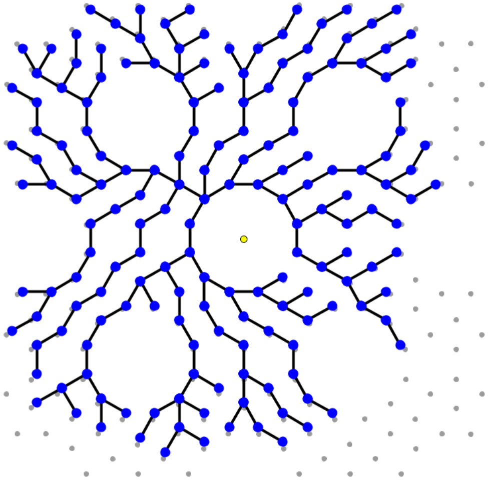

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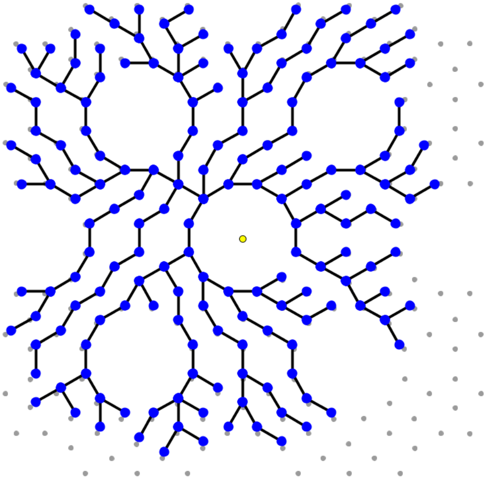

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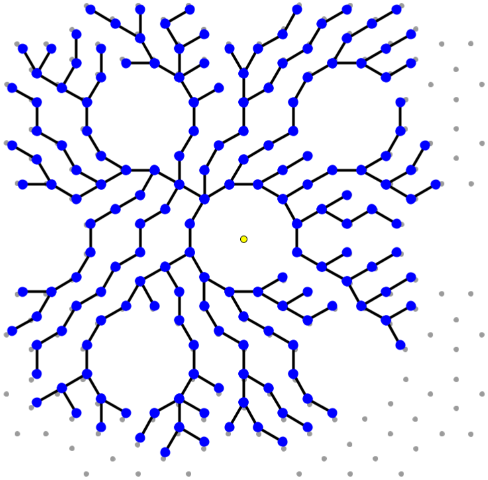

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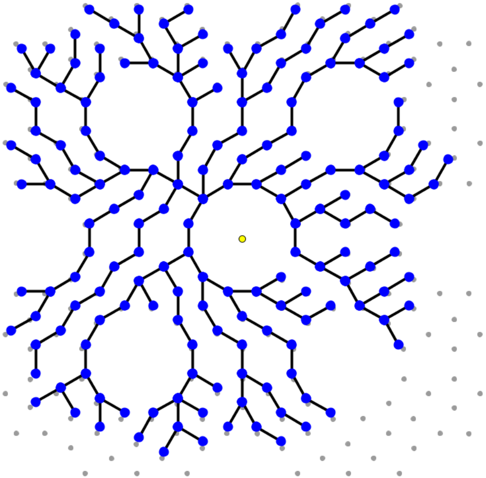

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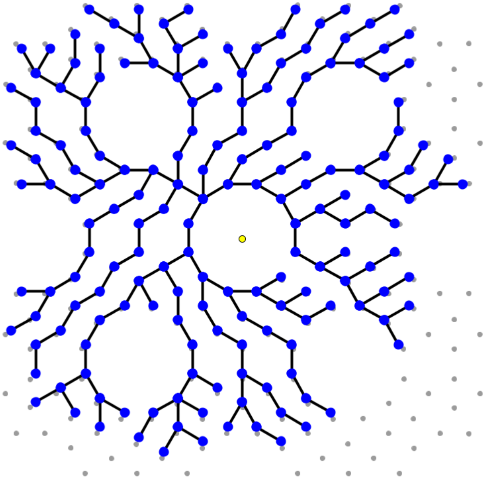

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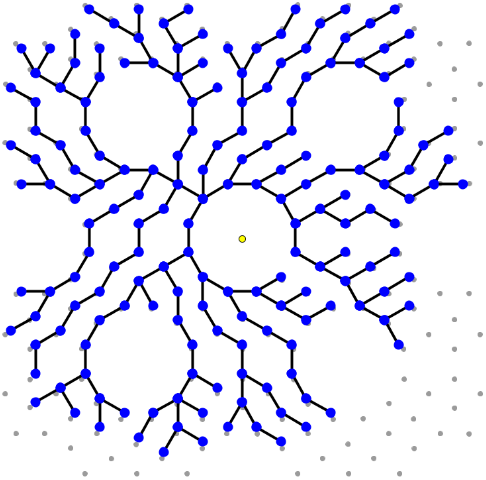

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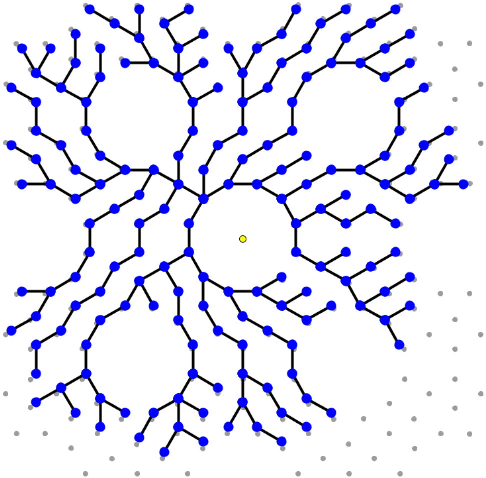

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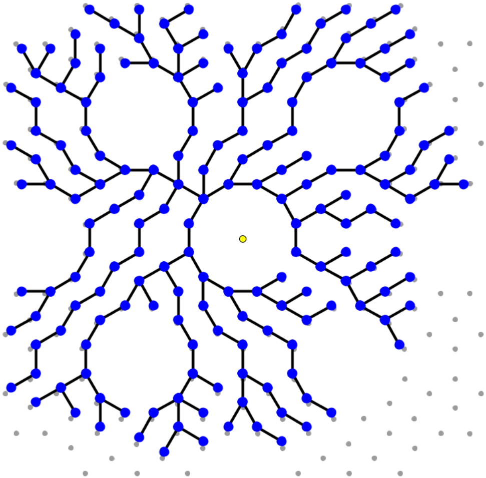

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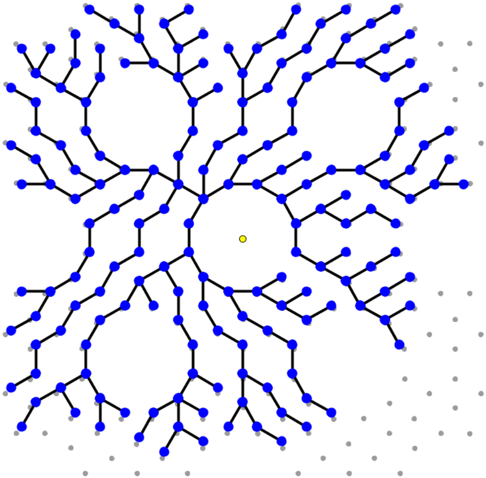

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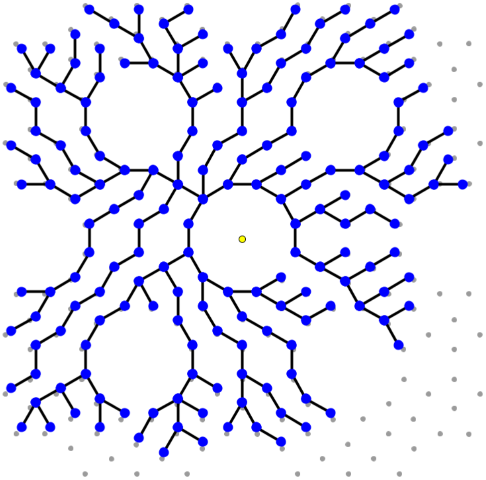

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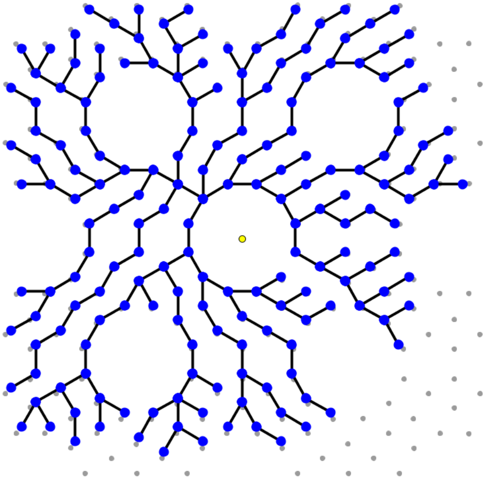

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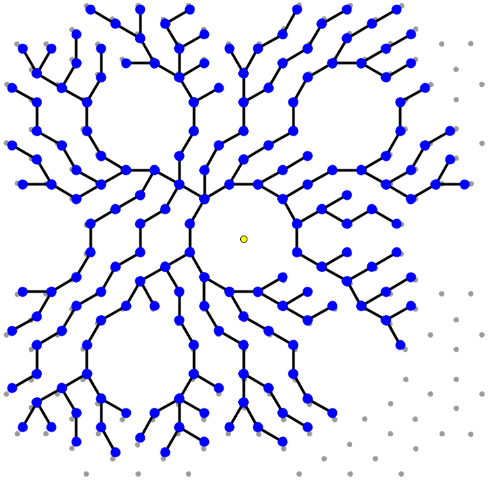

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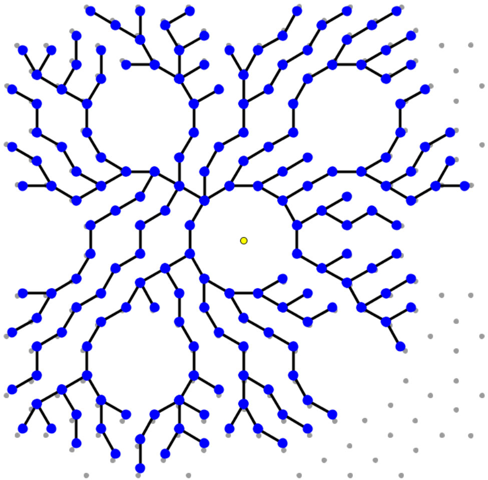

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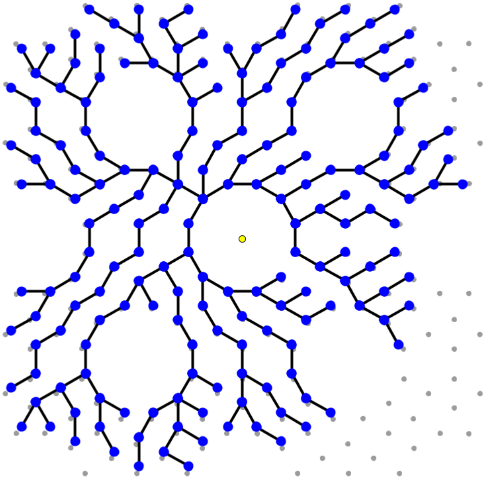

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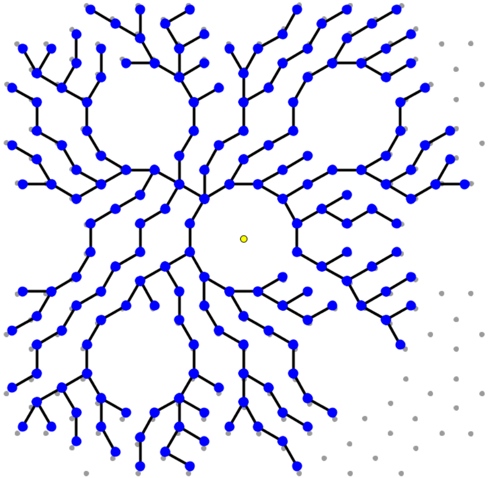

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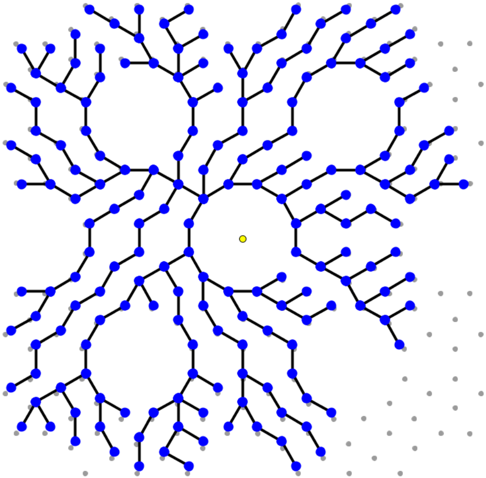

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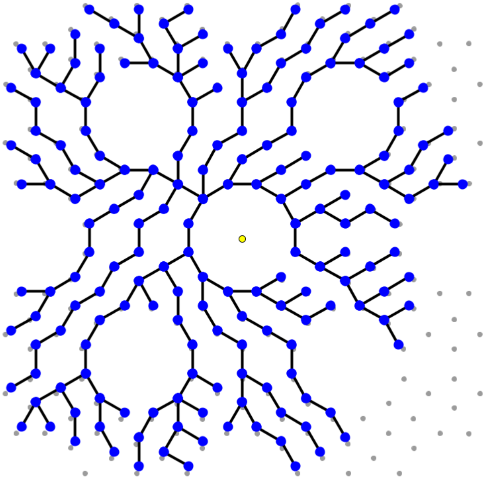

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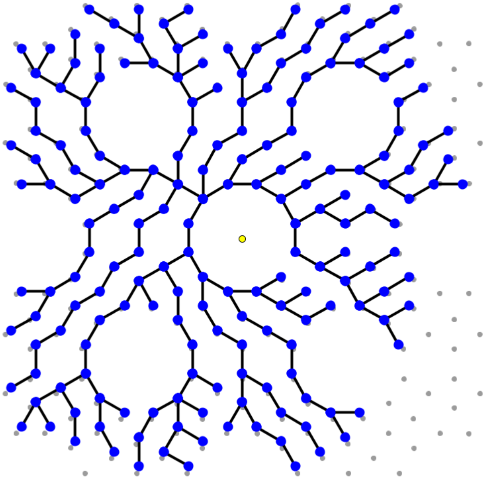

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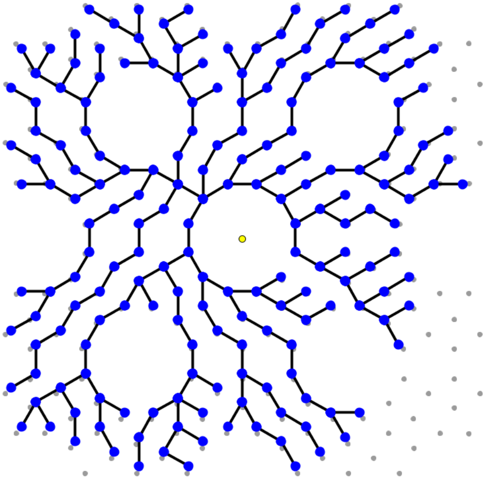

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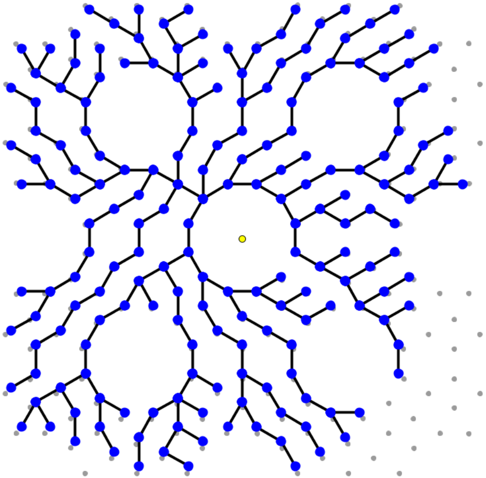

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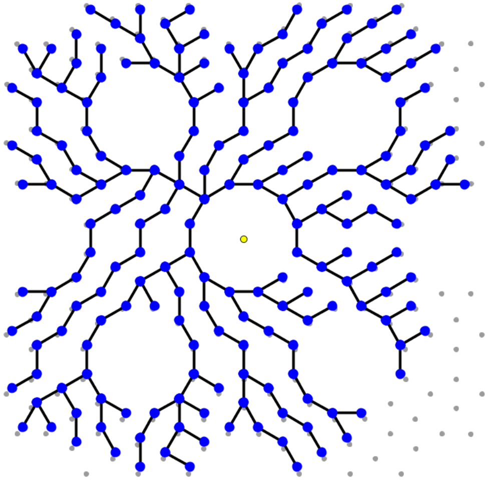

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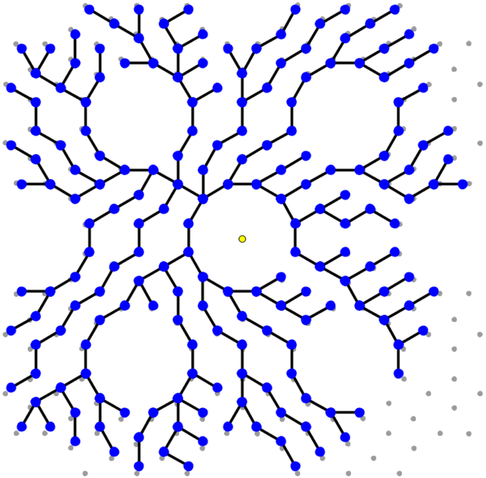

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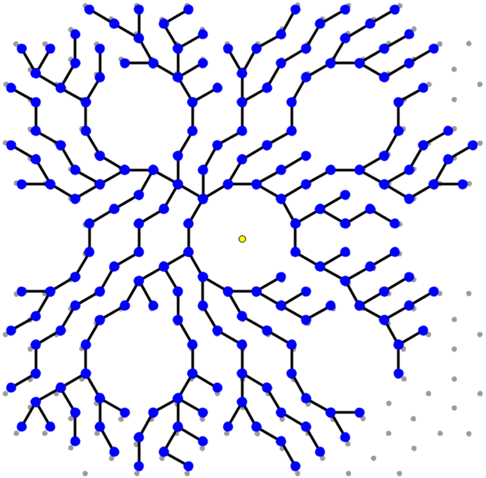

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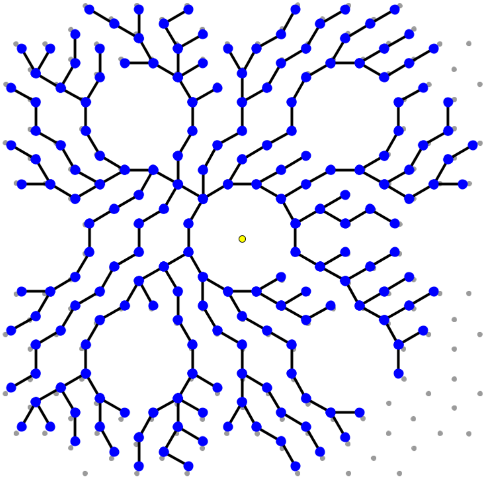
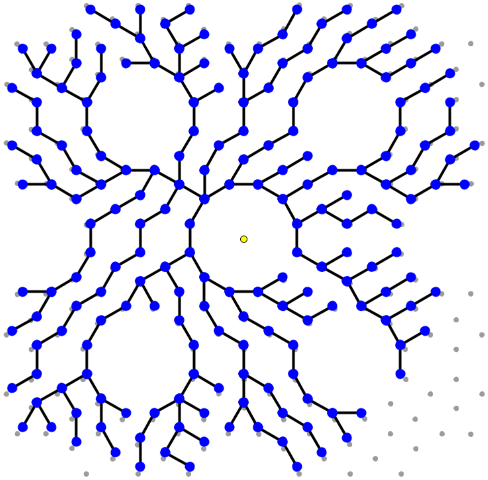


Automatic acquisition from tiny point cloud images



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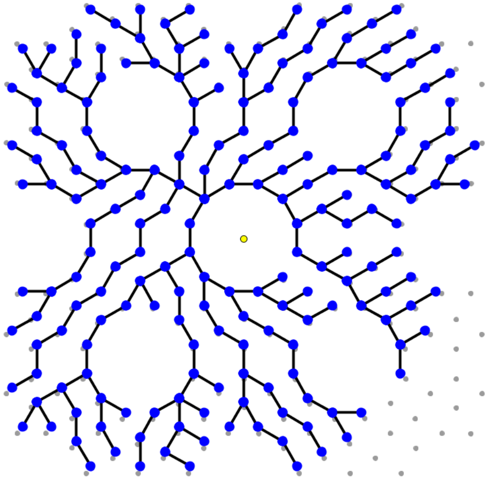

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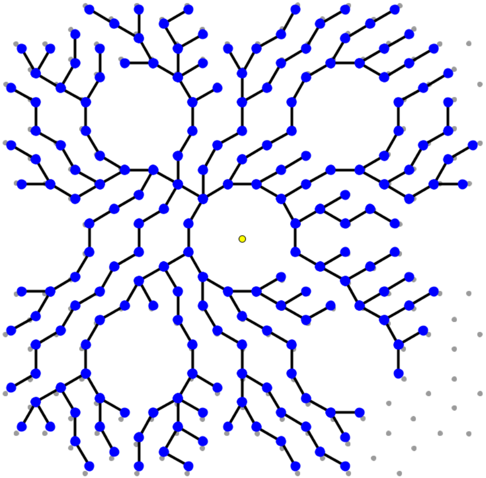

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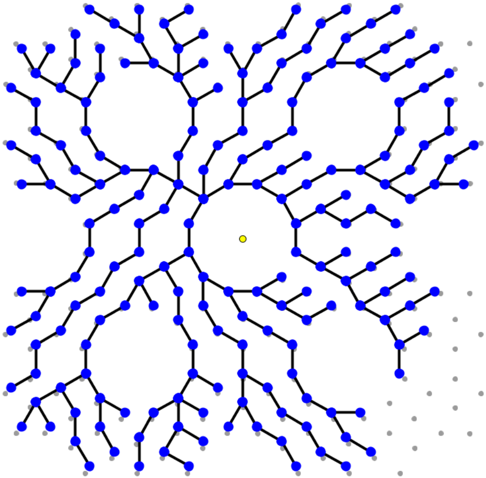

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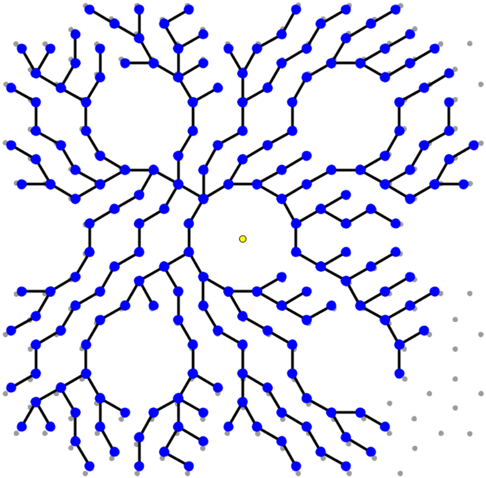

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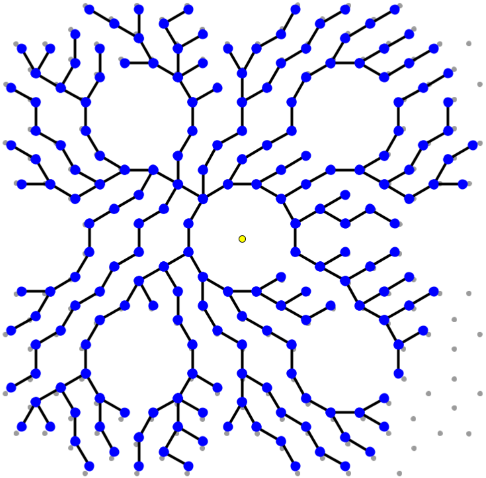

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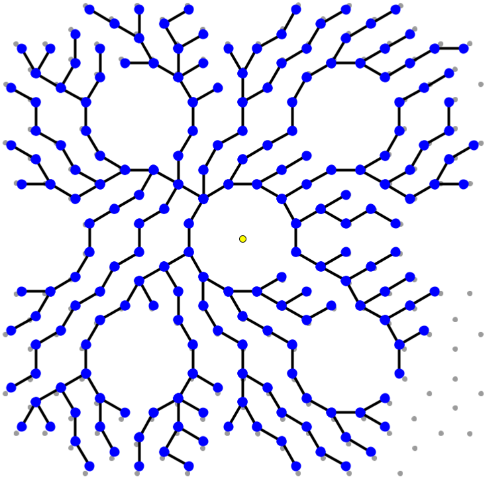

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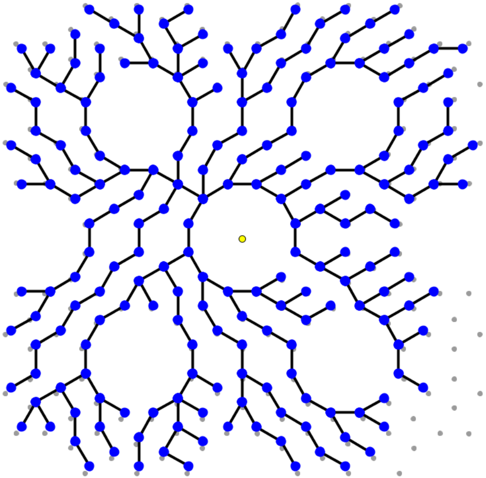

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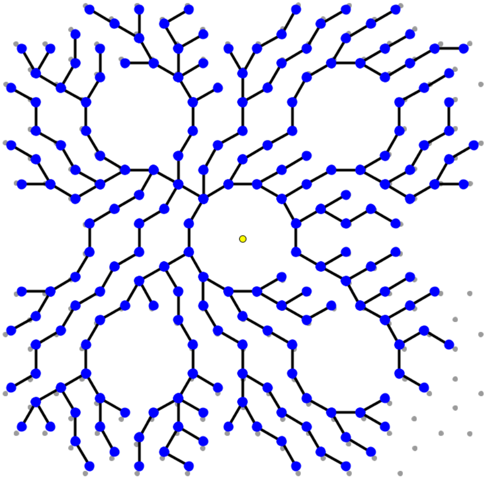

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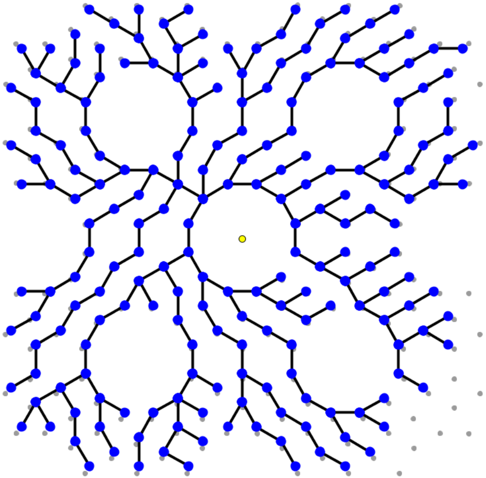

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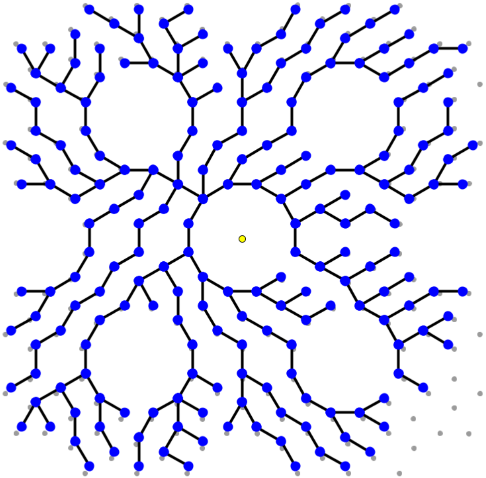

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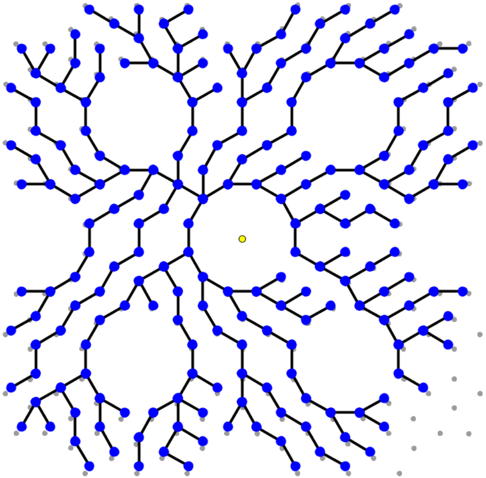

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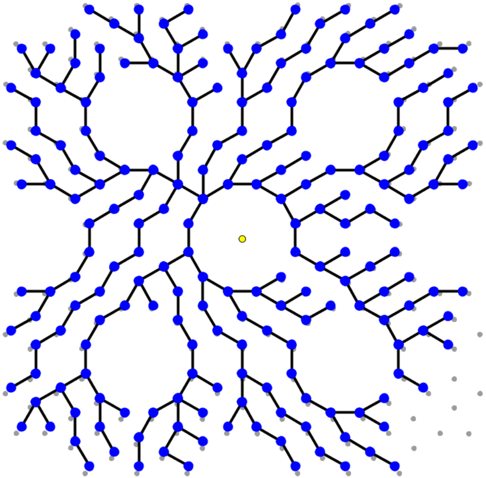

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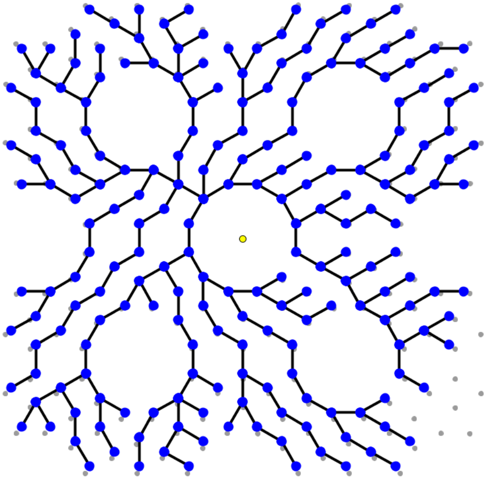

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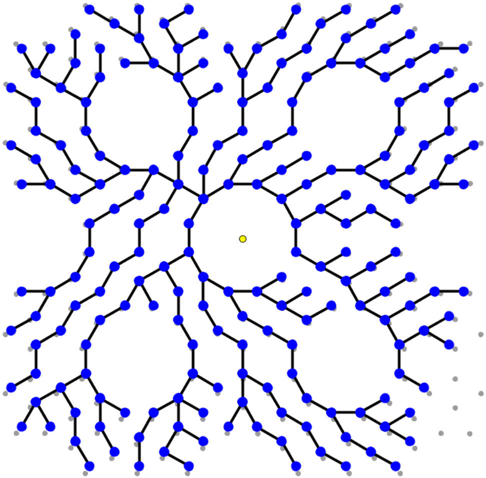

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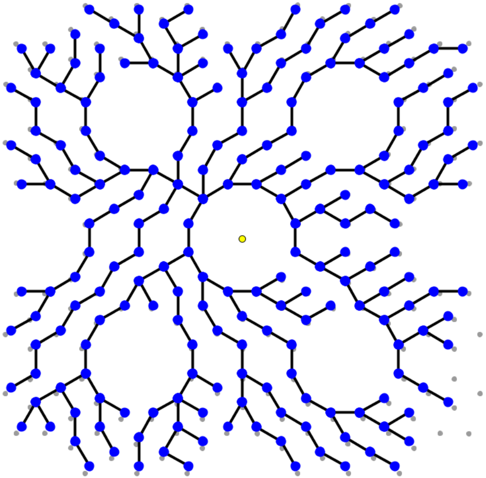

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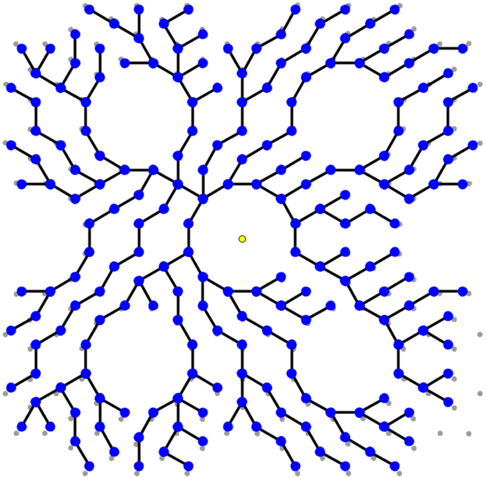

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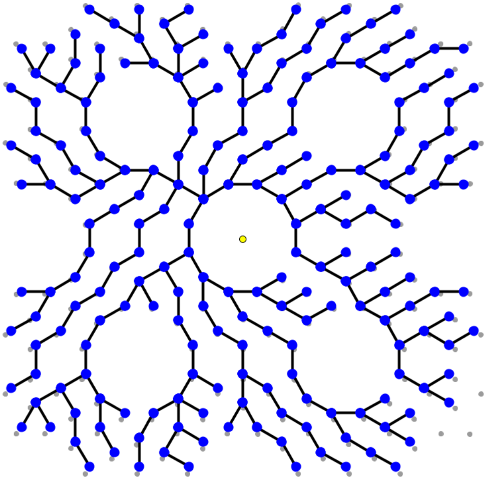

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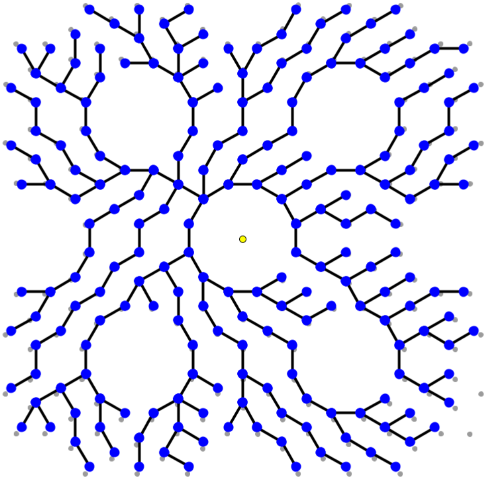

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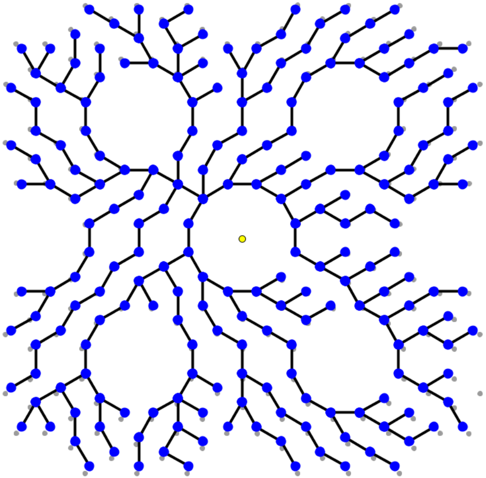

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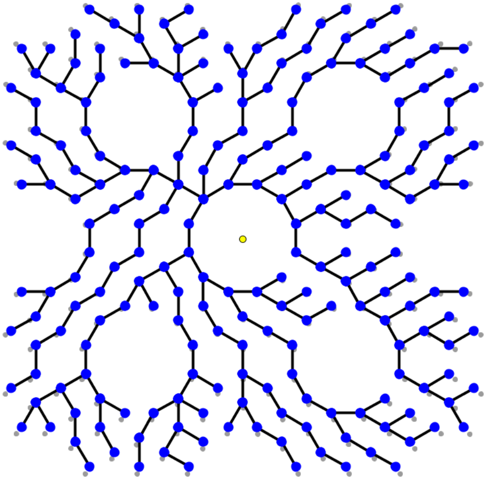

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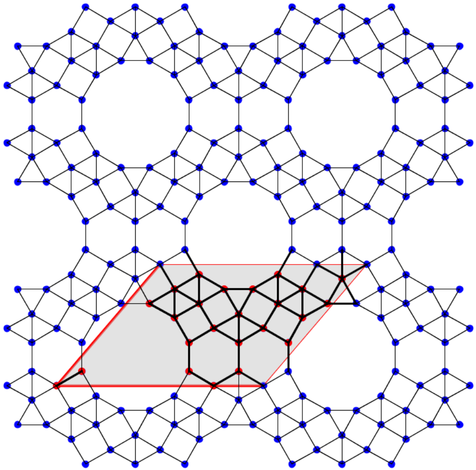
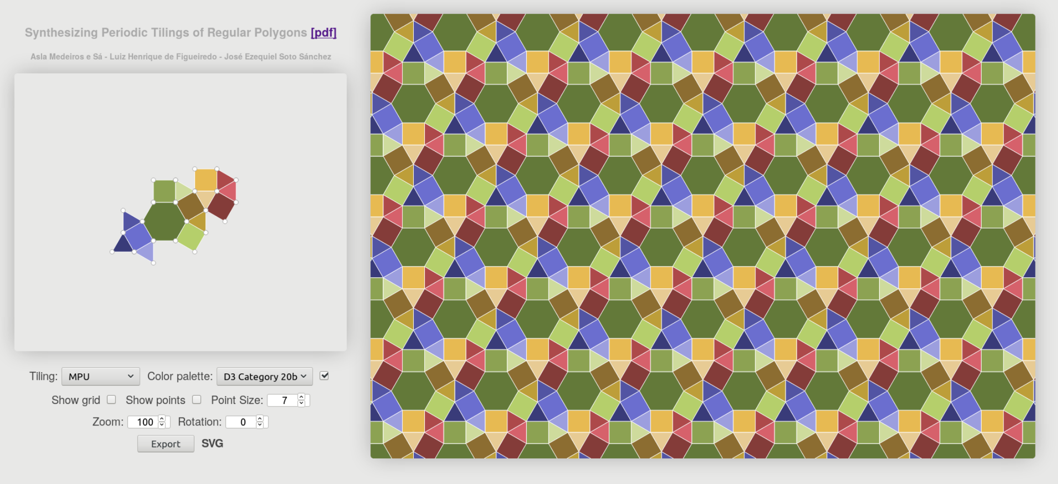
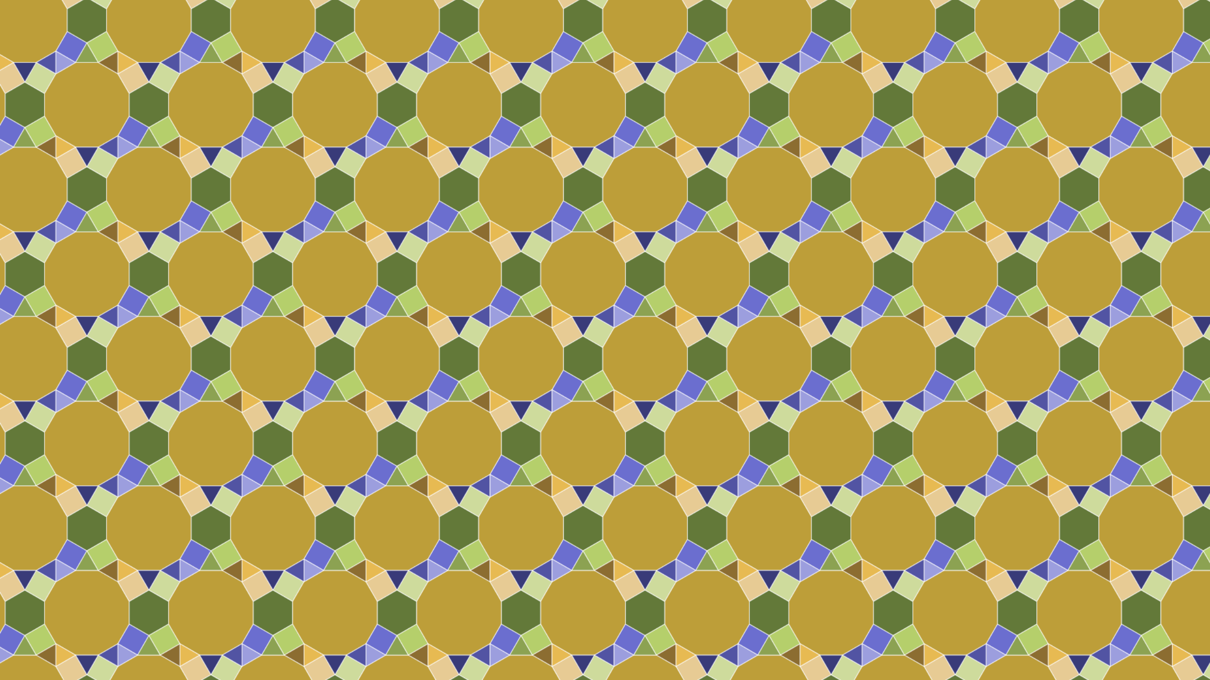
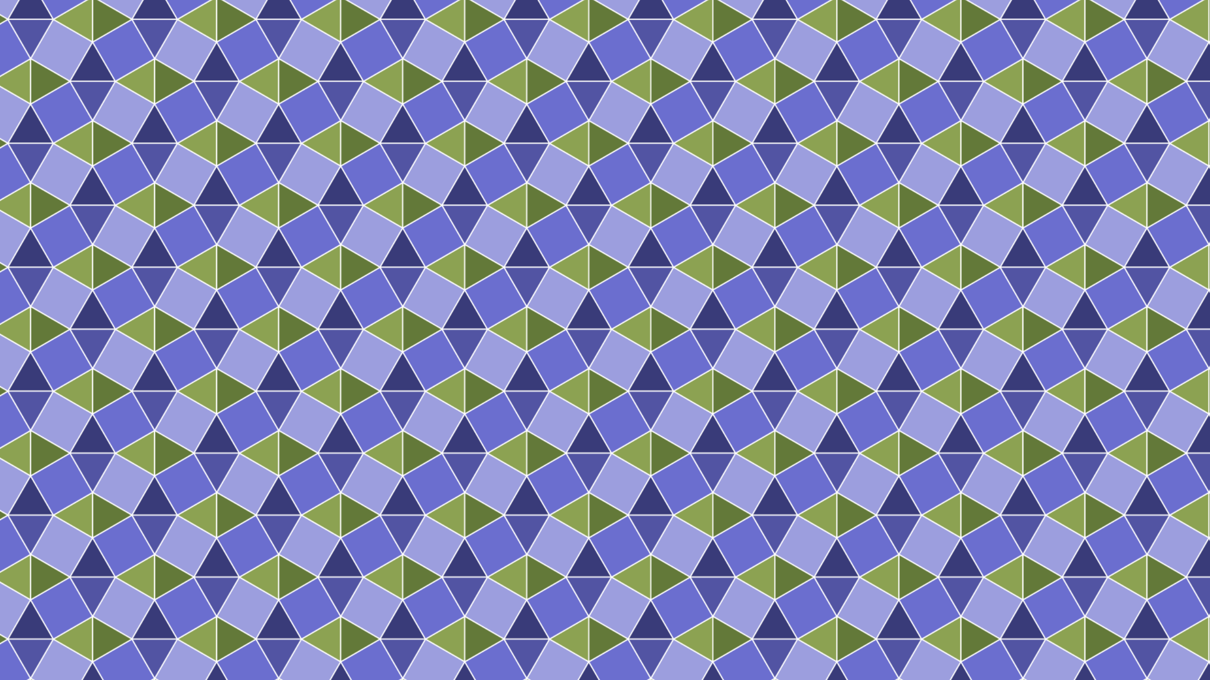
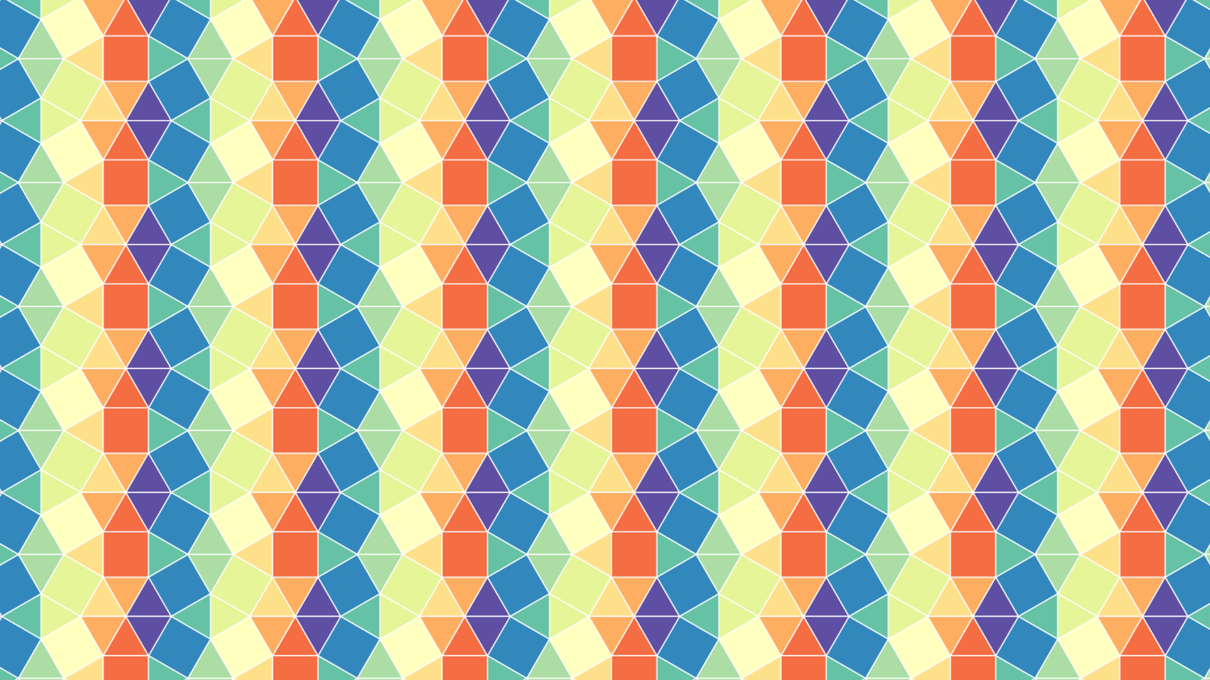


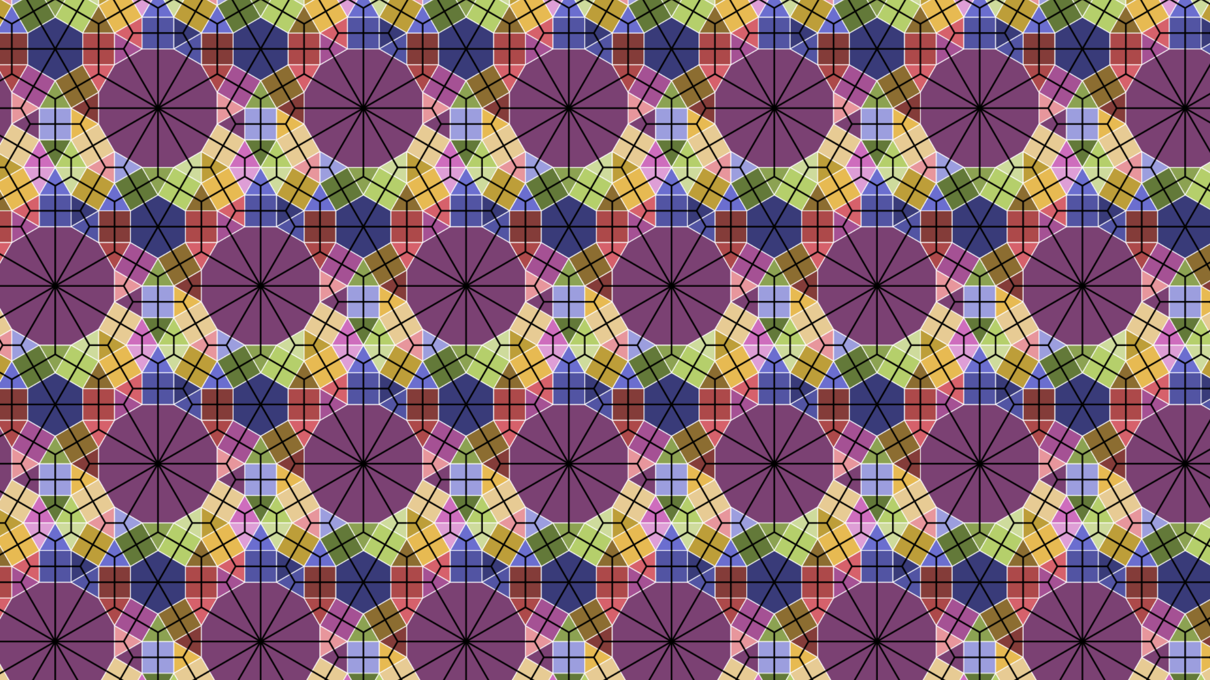
Web interface to catalogue

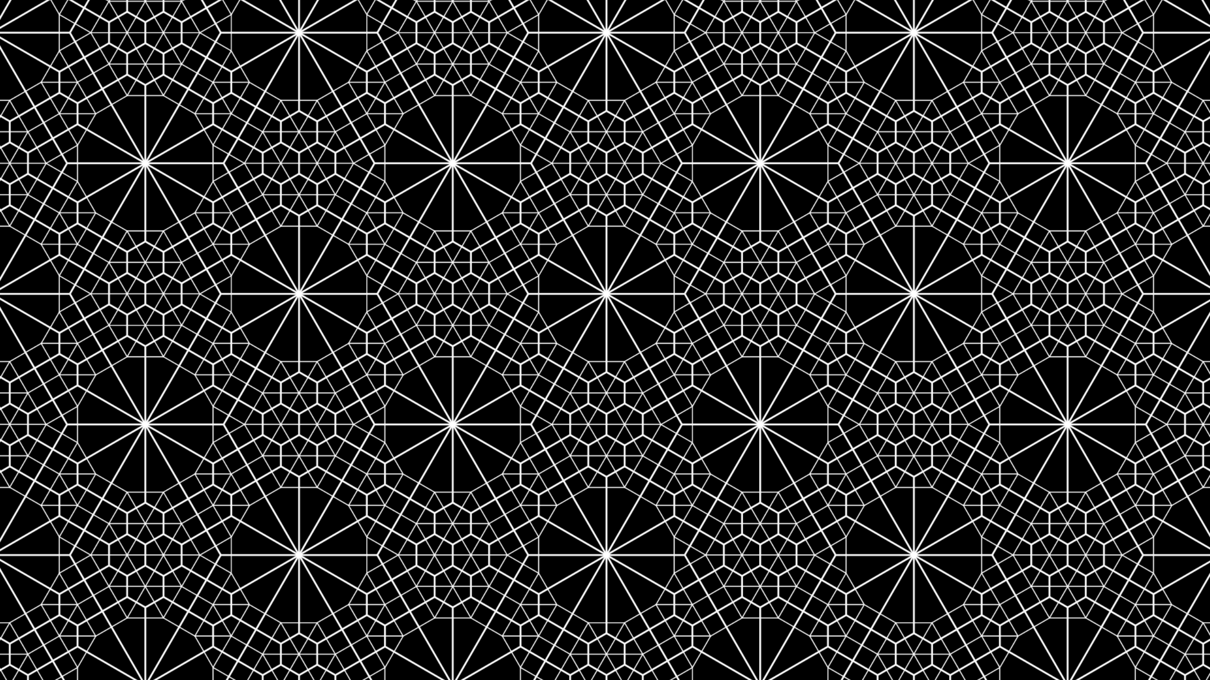












Web interface to catalogue

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