# SIBGRAPIE

## Beam casting implicit surfaces on the GPU with interval arithmetic

Francisco Ganacim Luiz Henrique de Figueiredo Diego Nehab



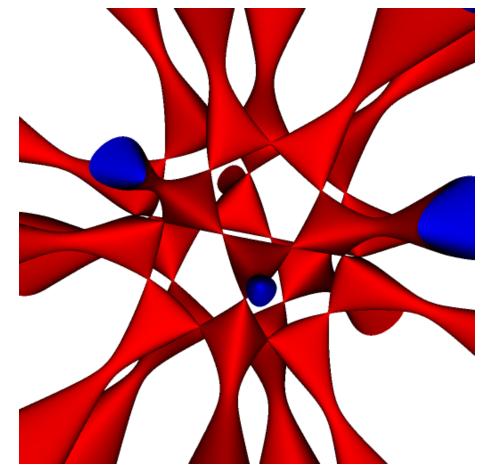






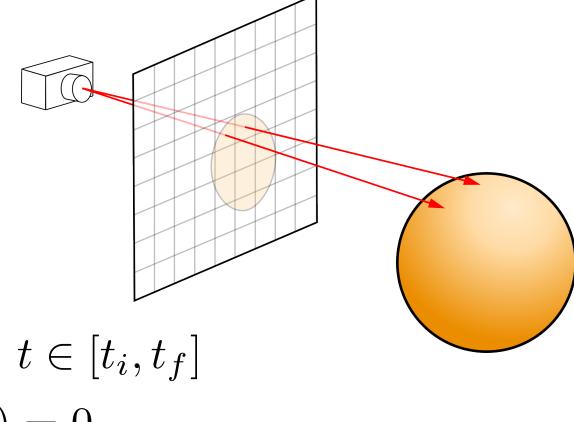
#### Related work

$$f: \mathbf{R}^3 \to \mathbf{R}$$
$$S = f^{-1}(0)$$
$$n_p = \frac{\nabla f(p)}{\|\nabla f(p)\|}$$



Barth:  $4(c^2x^2 - y^2)(c^2y^2 - z^2)(c^2z^2 - x^2) - (1 + 2c)(x^2 + y^2 + z^2 - 1)^2 = 0$ 

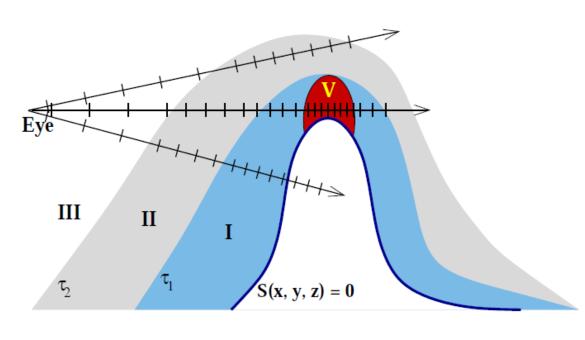
Ray casting



$$r(t) = p + tw, t \in [t_i, t_f]$$

$$g(t) := f(r(t)) = 0$$

#### Related work





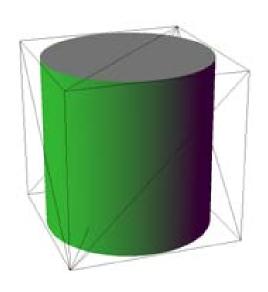


- Root finding by comparing consecutive samples along the ray
- GPU with GLSL

Singh et al. - 2009

#### Related work

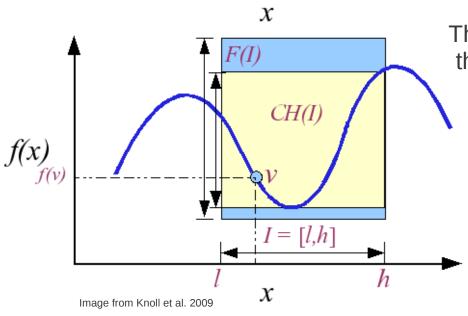




- Analytic solution finder for low-degree polynomial surfaces inside tetrahedrons
- GPU with HLSL

Loop & Blinn - 2006

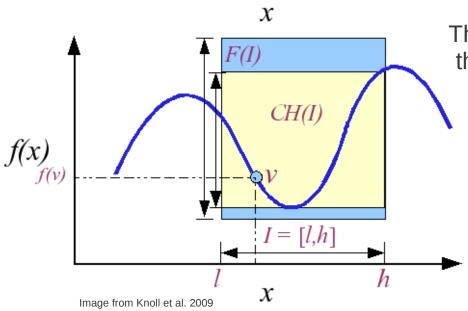
#### Interval arithmetic



The interval extension gives bounds for the range of a function on an interval

$$f(I) \subset F(I)$$

#### Interval arithmetic



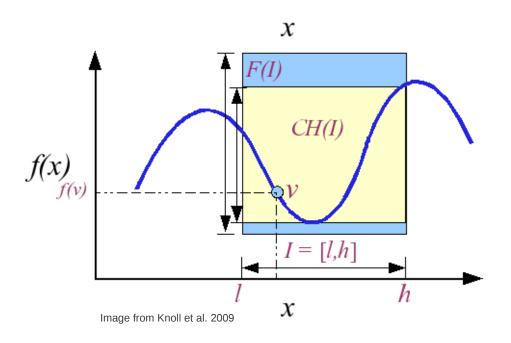
The interval extension gives bounds for the range of a function on an interval

$$f(I) \subset F(I)$$

root containment criteria

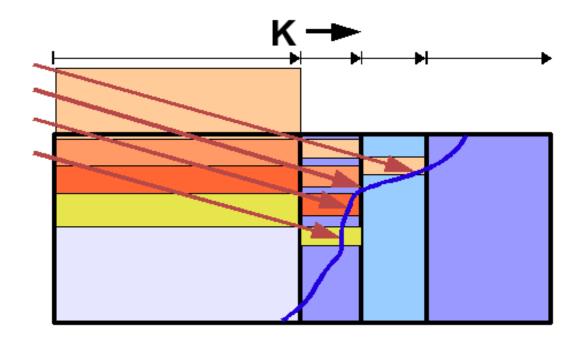
$$0 \notin F(I) \implies 0 \notin f(I)$$

#### Related work



- Mitchell 1990
- Cusatis et al. 1999

#### Related work

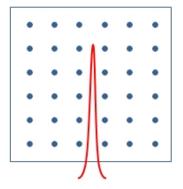


- Real-time ray casting on the CPU using SSE instructions
- Exploits ray coherence

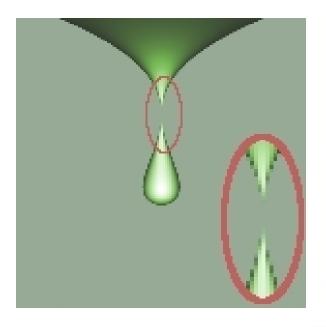
Knoll et al. - 2009

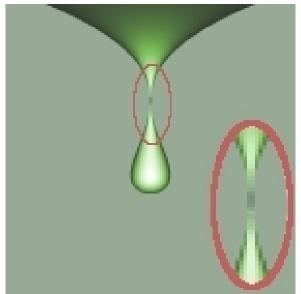


#### Limitations of point sampling



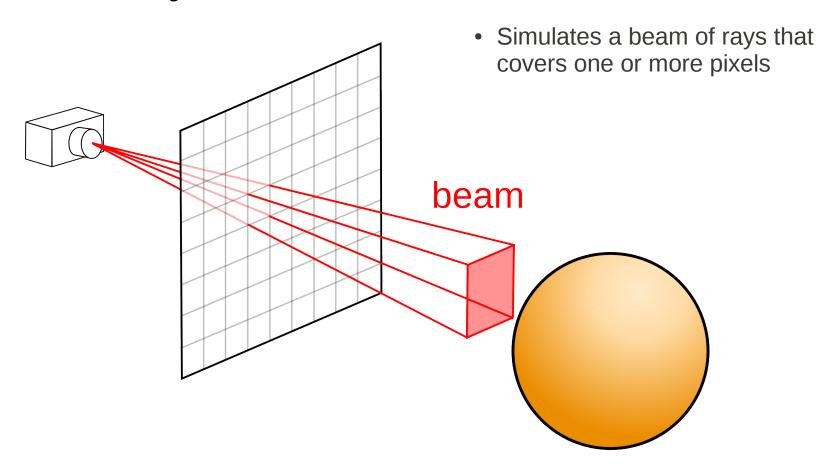
- A low sampling rate is necessary to achieve real-time frame rates
- Thin features may be missed





Images from Flórez, 2008

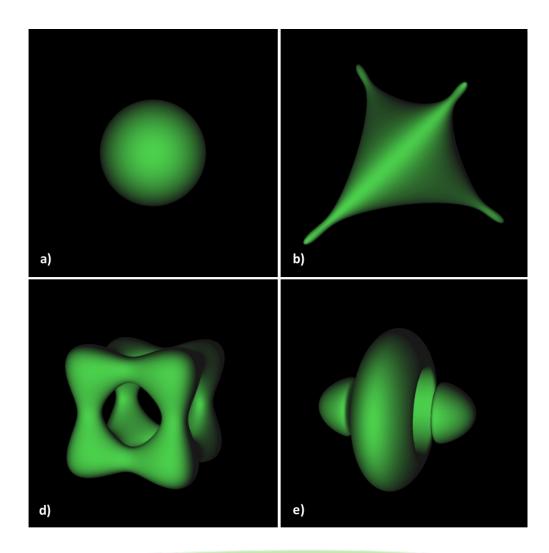
#### Beam casting





#### Related work

 Beam casting with space subdivision on the CPU



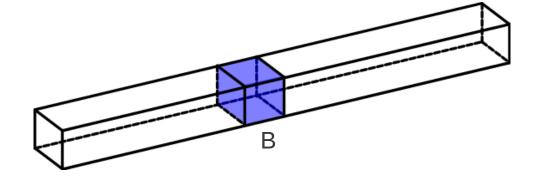
Flórez - 2008 Flórez et al. - 2009



- Beam casting
- GPU CUDA
- Space subdivision
- Anti-aliasing



#### root containment test

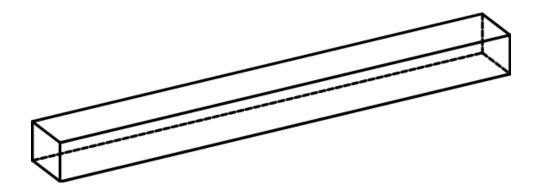


$$f(B) \subset F(B)$$

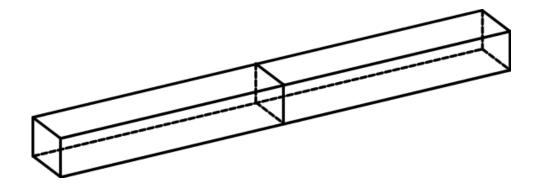
root containment criteria

$$0 \notin F(B) \implies 0 \notin f(B)$$

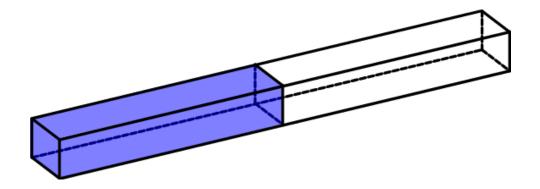
- Depth-first search on a binary tree, where the nodes are generated by bisecting the input beam
- Finish when a "small" block is found



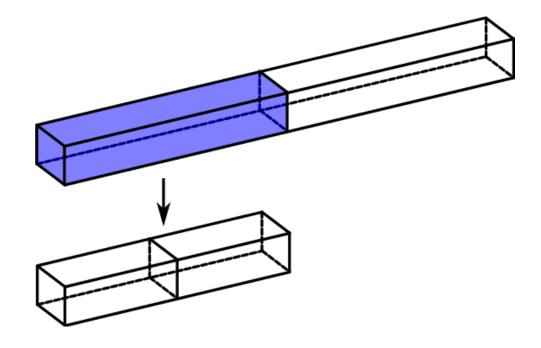
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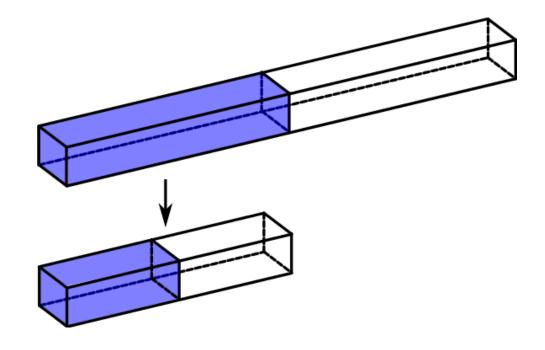
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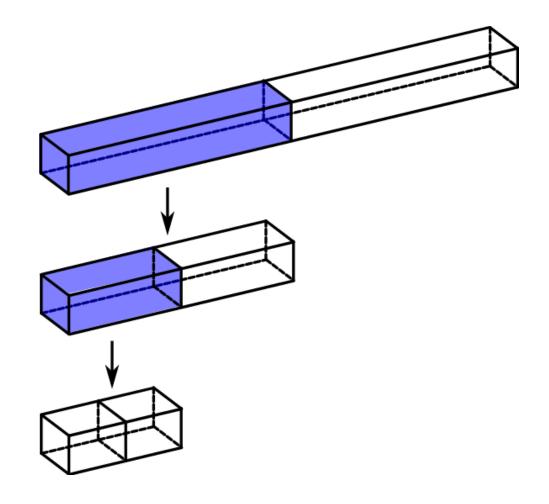
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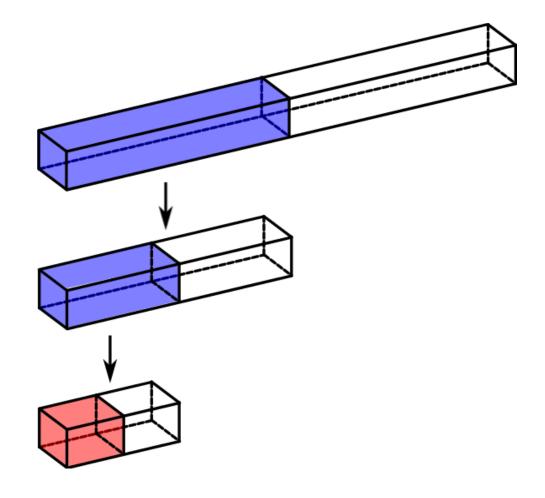
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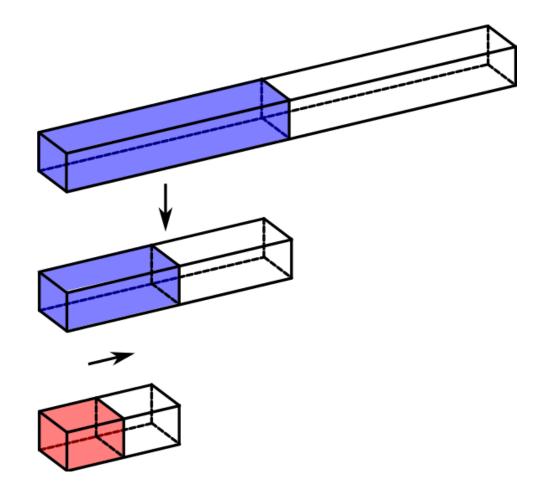
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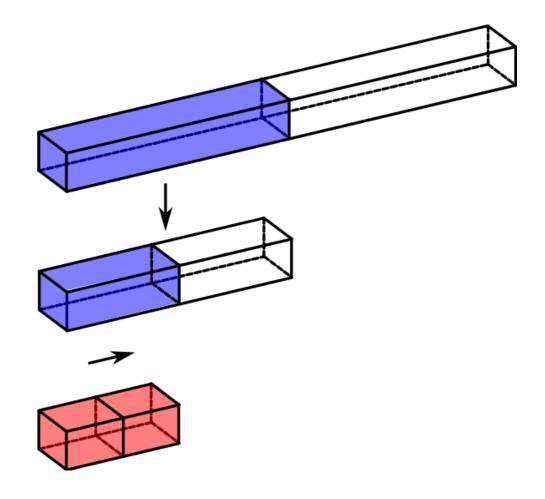
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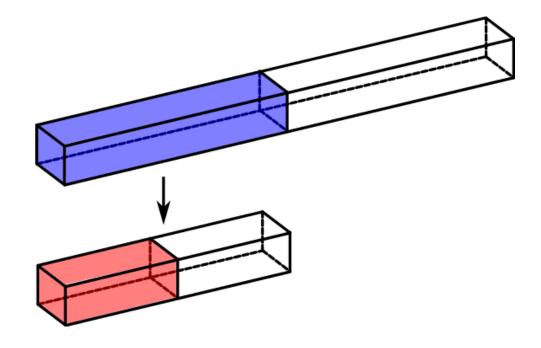
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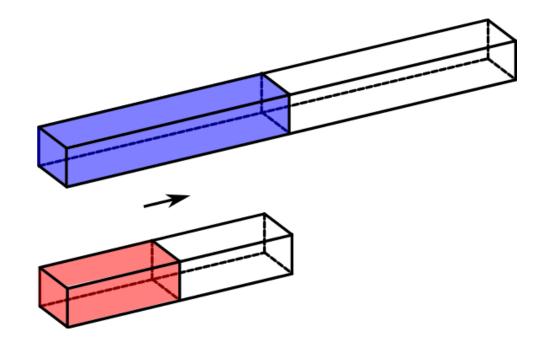
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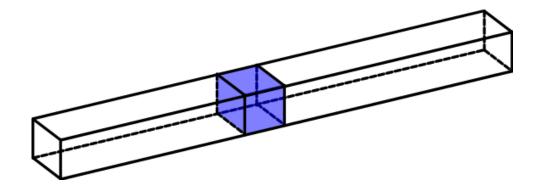
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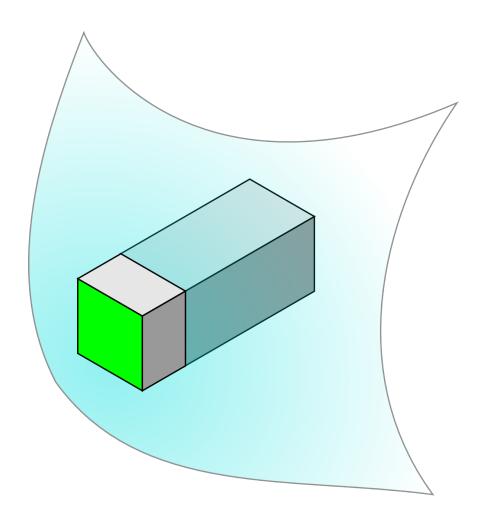


- Depth-first search on a binary tree, where the nodes are generated by bisecting the input beam
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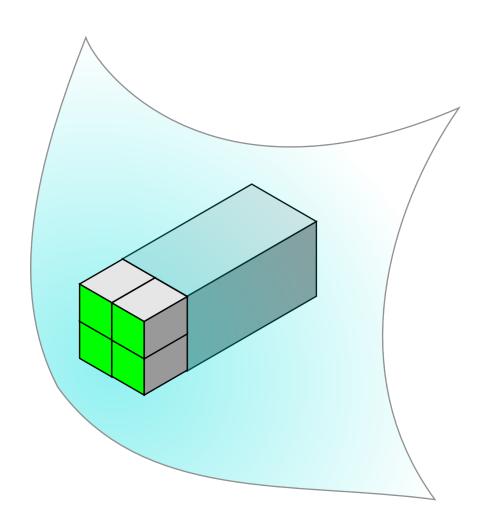
#### Bisection precision limit

 The geometry of the beam imposes a limit to the precision



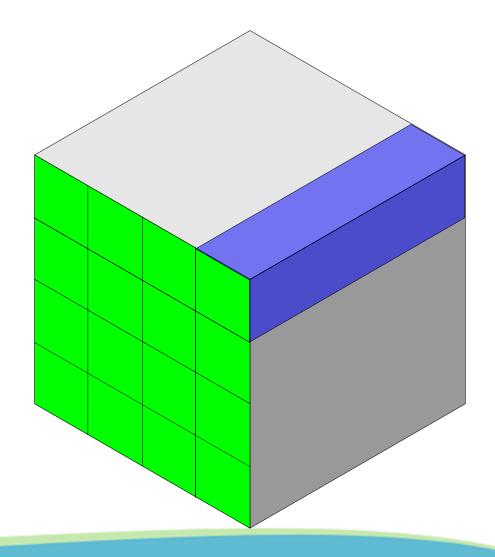
#### Bisection precision limit

- The geometry of the beam imposes a limit to the precision
- We need to subdivide the beam to achieve more precision



Spatial subdivision

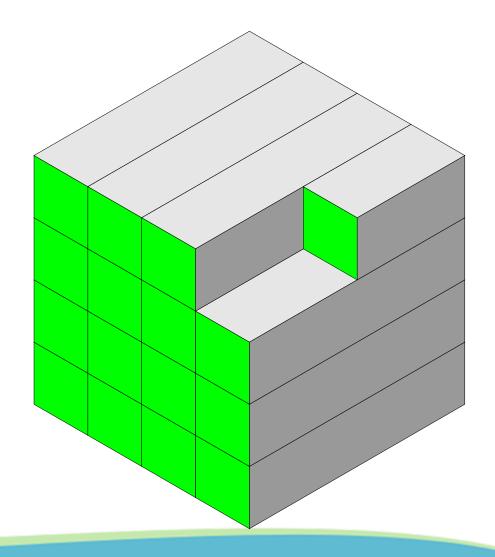
1. Large beams are cast





Spatial subdivision

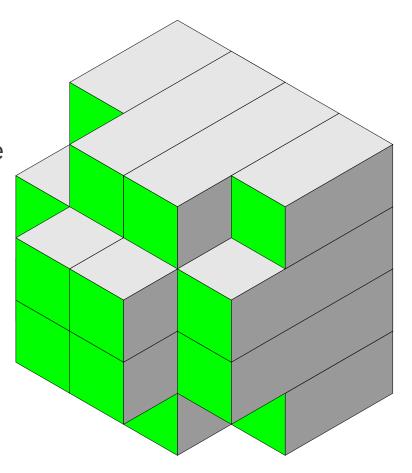
1. Large beams are cast





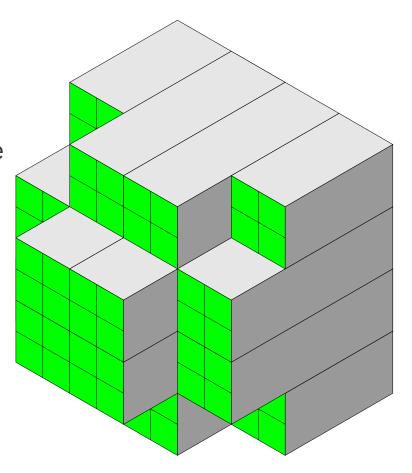
#### Spatial subdivision

- 1. Large beams are cast
- 2. We find an approximation of the surface

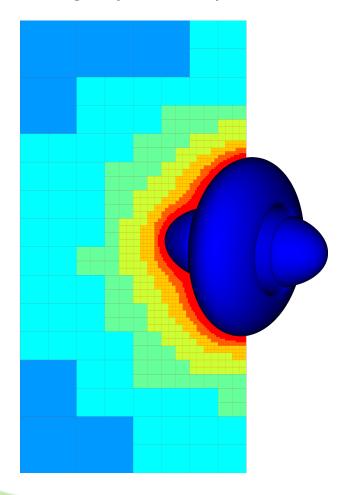


#### Spatial subdivision

- 1. Large beams are cast
- 2. We find an approximation of the surface
- 3. The remaining beams are subdivided



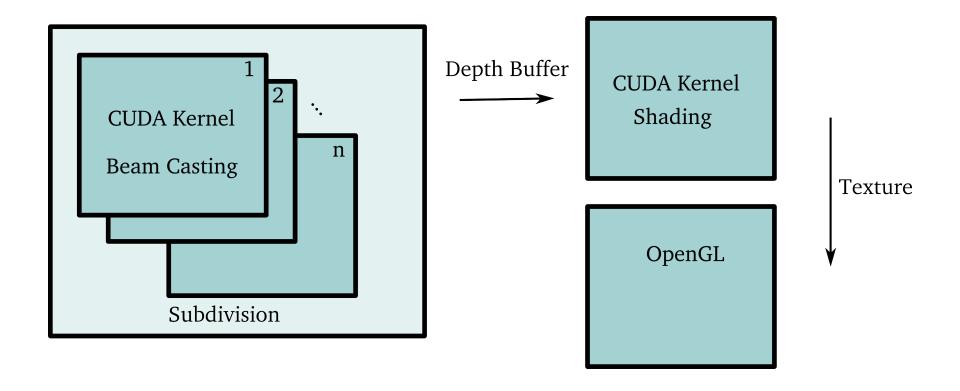
#### Image space adaptation



- Regions far from the surface are eliminated by large beams
- Regions near the surface require more accuracy and therefore smaller beams

## Implementation

architecture

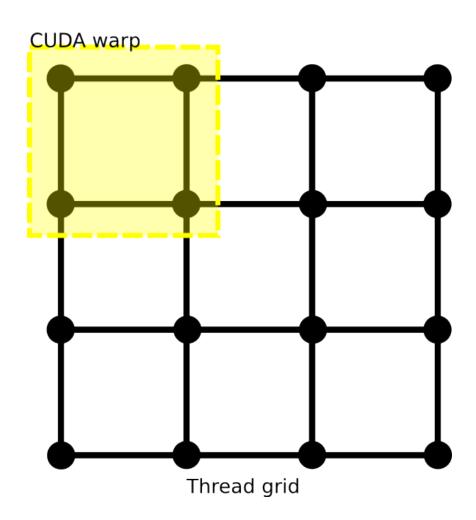




## Implementation

#### Casting step

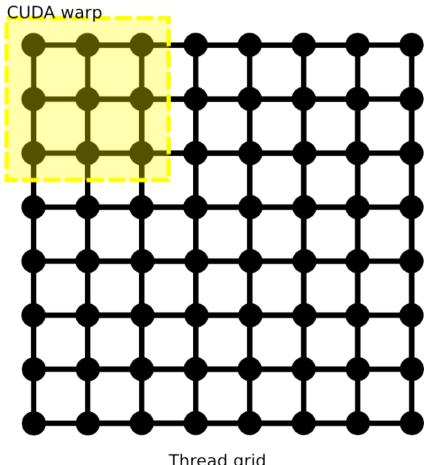
- In each step, the beams are arranged on a grid
- Each beam is processed by one CUDA thread
- The threads are grouped in warps by their geometric proximity to reduce divergence



## **Implementation**

#### Casting step

- The result of one step is saved in a memory buffer
- In the next step, the buffer is processed by the new grid

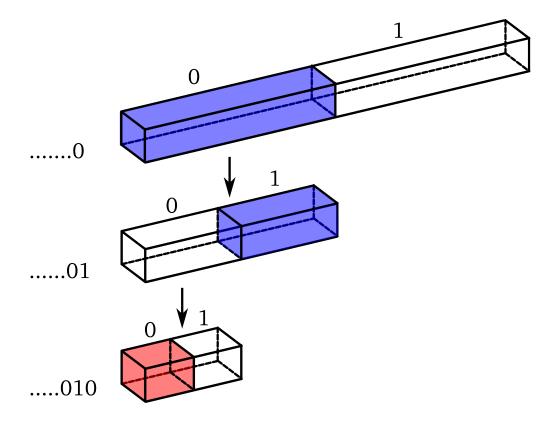


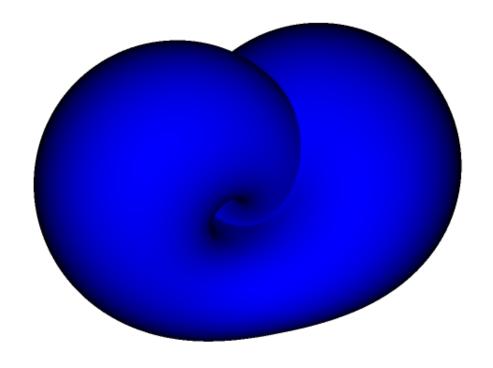
Thread grid

#### Implementation

#### Bisection kernel

 To avoid using recursion or an explicit stack, the bisection kernel stores the search status in one integer



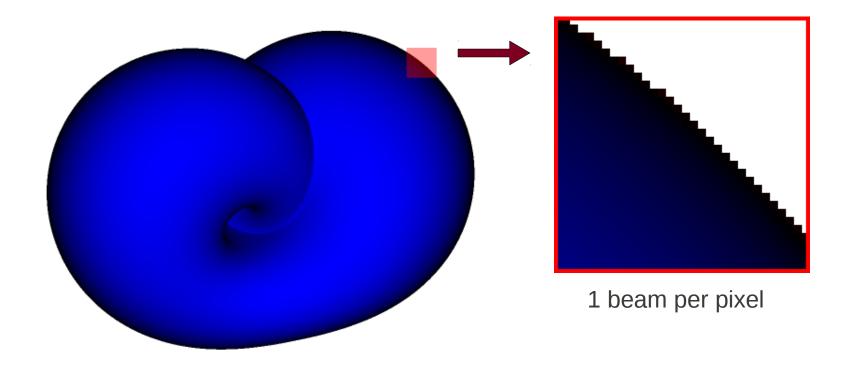


	1 beam per pixel	4 beams per pixel
512 <sup>2</sup> pixels	362	133
1024 <sup>2</sup> pixels	131	41

performance in  $\mathit{fps}$ 

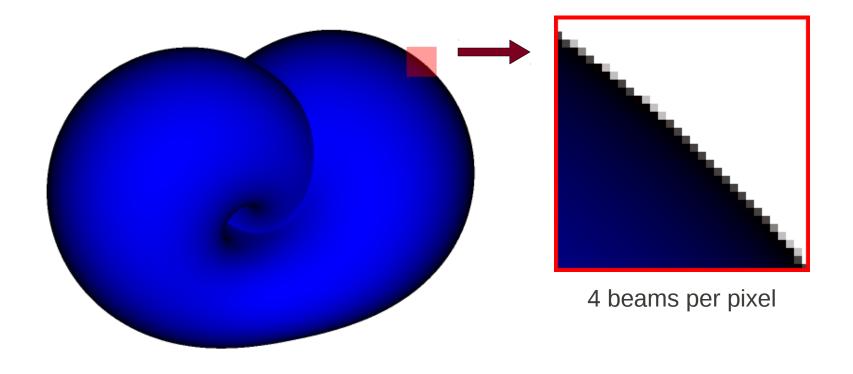
Klein: 
$$(x^2 + y^2 + z^2 + 2y - 1)((x^2 + y^2 + z^2 - 2y - 1)^2 - 8z^2) + 16xz(x^2 + y^2 + z^2 - 2y - 1)$$





Klein: 
$$(x^2 + y^2 + z^2 + 2y - 1)((x^2 + y^2 + z^2 - 2y - 1)^2 - 8z^2) + 16xz(x^2 + y^2 + z^2 - 2y - 1)$$



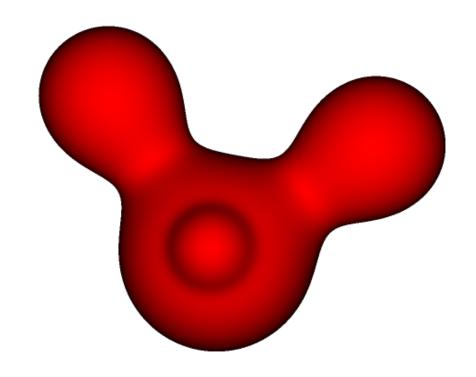


Klein: 
$$(x^2 + y^2 + z^2 + 2y - 1)((x^2 + y^2 + z^2 - 2y - 1)^2 - 8z^2) + 16xz(x^2 + y^2 + z^2 - 2y - 1)$$



	1 beam per pixel	4 beams per pixel
512 <sup>2</sup> pixels	1322	570
1024² pixels	545	182

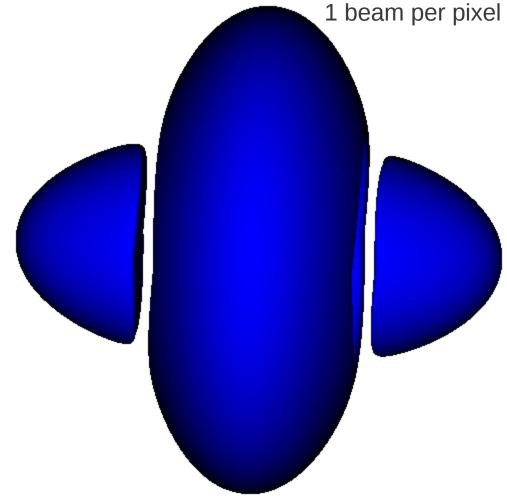
performance in fps



Blob (non-algebraic)

	1 beam per pixel	4 beams per pixel
512 <sup>2</sup> pixels	240	72
1024 <sup>2</sup> pixels	70	26

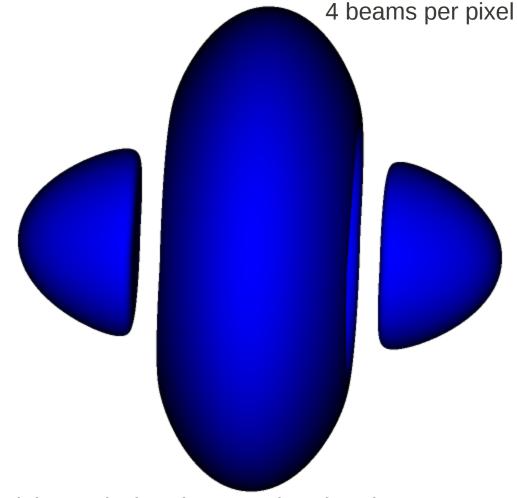
performance in fps



Mitchell: 
$$4(x^4 + (y^2 + z^2)^2 + 17x^2(y^2 + z^2)) - 20(x^2 + y^2 + z^2) + 17x^2(y^2 + z^2)$$

	1 beam per pixel	4 beams per pixel
512 <sup>2</sup> pixels	240	72
1024² pixels	70	26

performance in fps



Mitchell: 
$$4(x^4 + (y^2 + z^2)^2 + 17x^2(y^2 + z^2)) - 20(x^2 + y^2 + z^2) + 17x^2(y^2 + z^2)$$

#### Conclusion

- Simple method
- Easy to implement
- Achieves real-time frame rates
- Anti-aliasing



#### **Future directions**

- Affine arithmetic
- Better load balancing
- Investigate the role of thread divergence



### Thank you!



# SIBGRAPIE

## Beam casting implicit surfaces on the GPU with interval arithmetic

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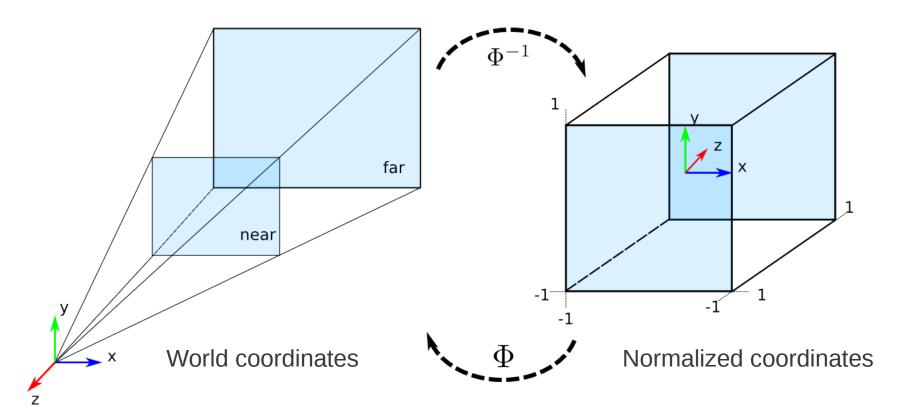






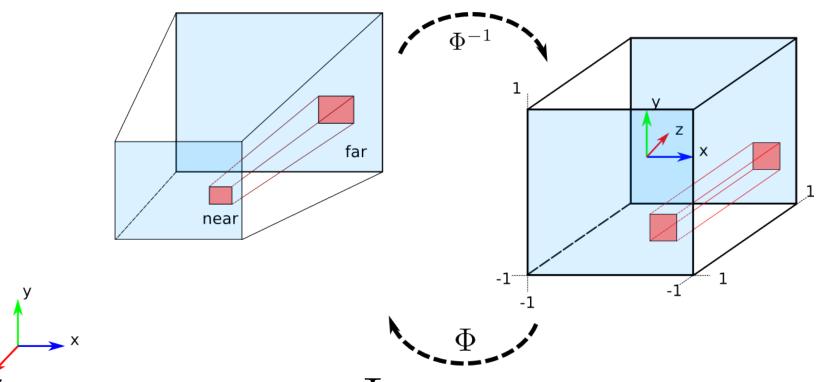
#### Our method

#### Projection transformation



#### Our method

#### Projection transformation



•The interval extension of  $\Phi$  provides bounds (a box) in world coordinates, to a given normalized volume