Approximating Implicit Curves on Plane and Surface Triangulations with Affine Arithmetic

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ICMC-USP

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Collaborators

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Overview

Problem Setup

Given a planar or surface triangulation $\mathcal{T}$ and $f : \mathbb{R}^d \to \mathbb{R}$, compute a *robust* adaptive polygonal approximation of the curve given implicitly by f on $\mathcal{T}$: $\mathcal{C} = \{\mathbf{x} \in \mathcal{T} : f(\mathbf{x}) = 0\}$.

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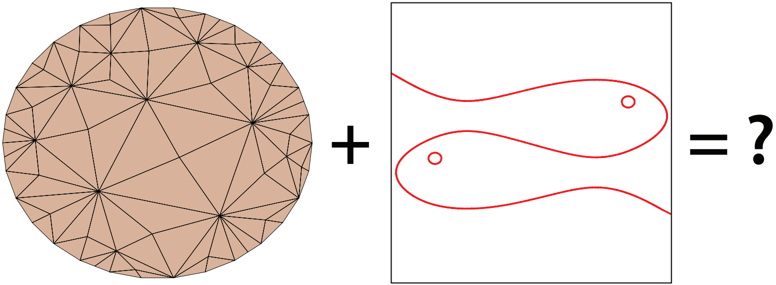

$\mathcal{C} = f^{-1}(0)$

Possible Solution?

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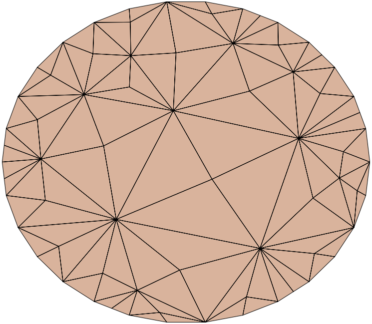

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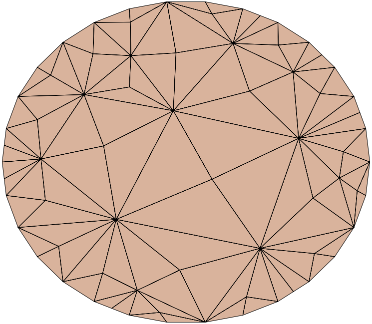

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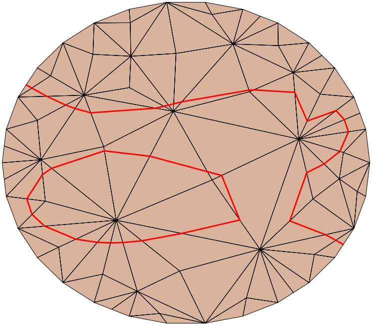

Marching Triangles

$\#\triangle = 101$

Possible Solution?

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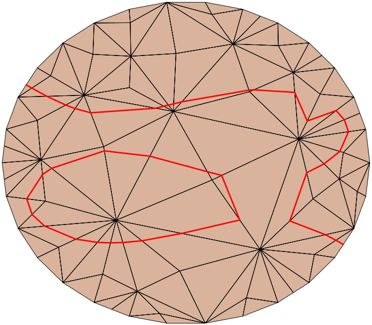

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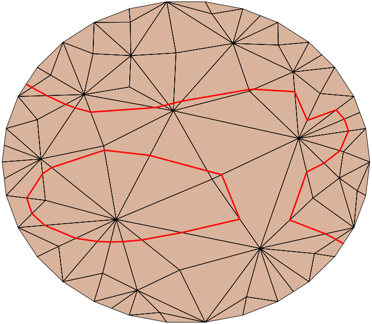

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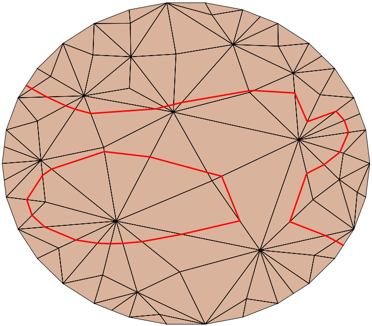

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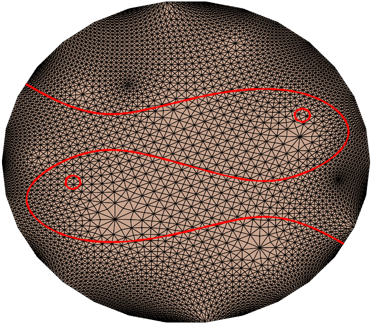

Marching Triangles

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Possible Solution?

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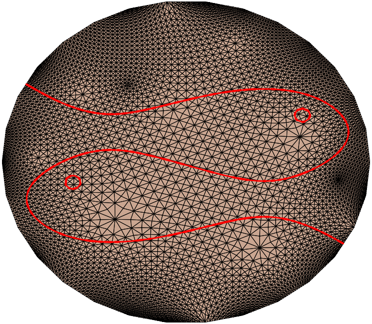

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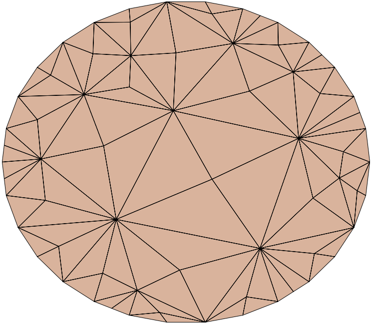
level 0



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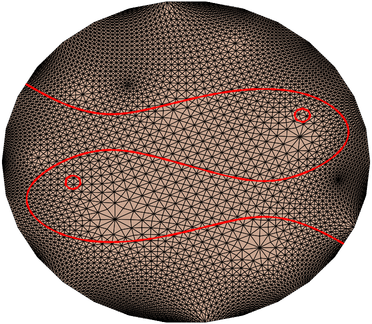

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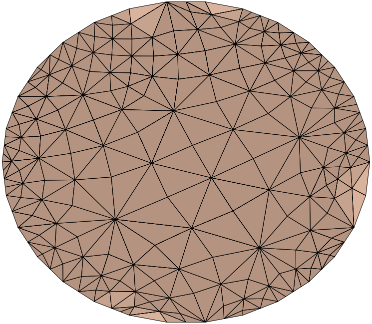

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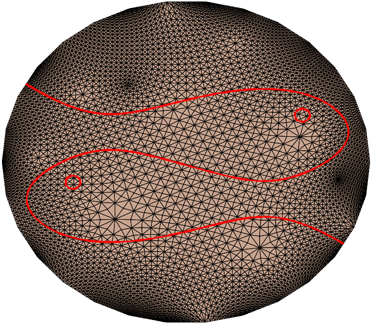

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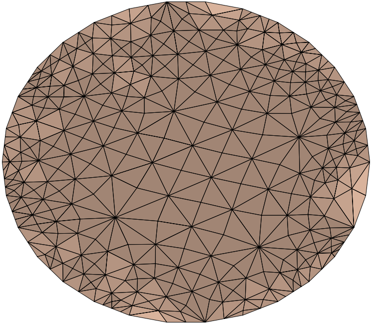
level 2



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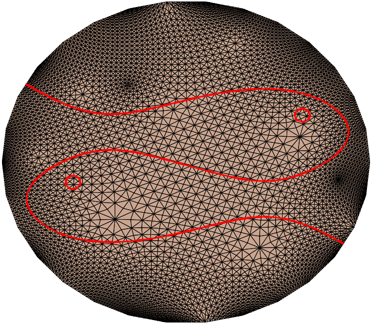

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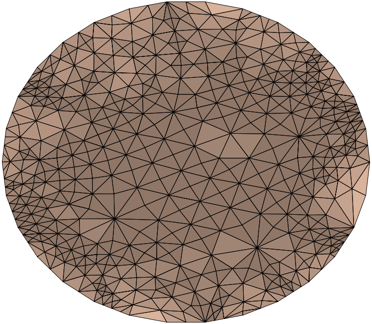

Our Method

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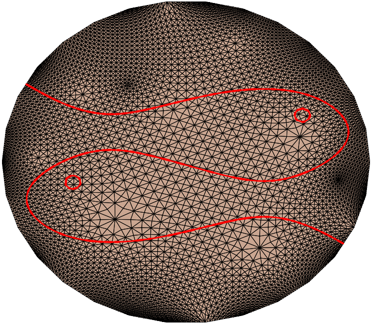

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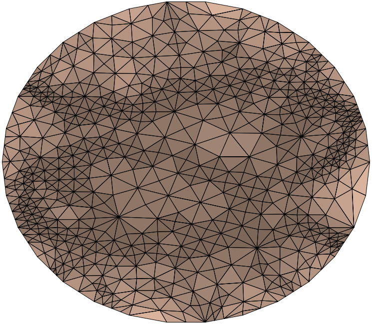

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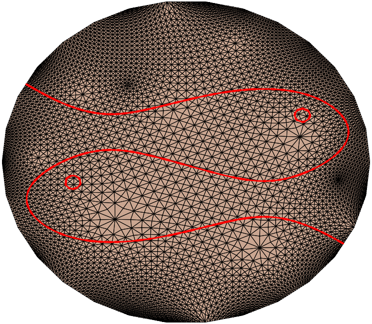

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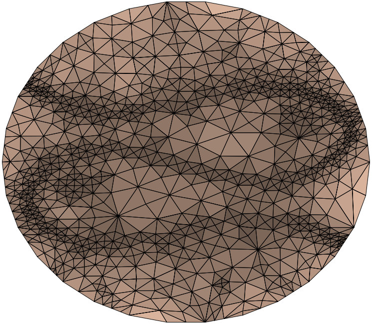

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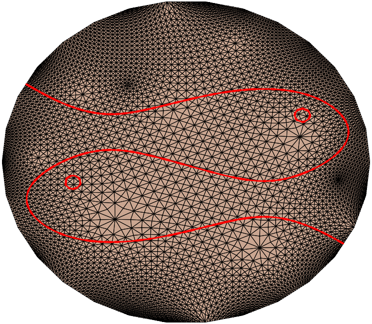

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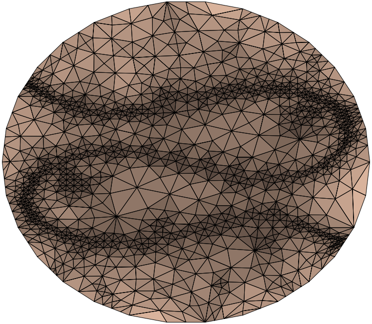
level 6



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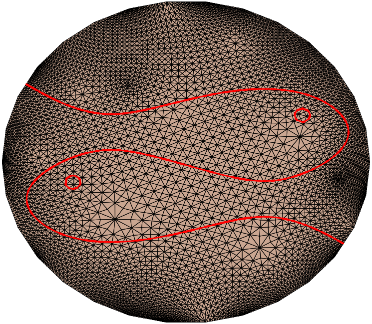

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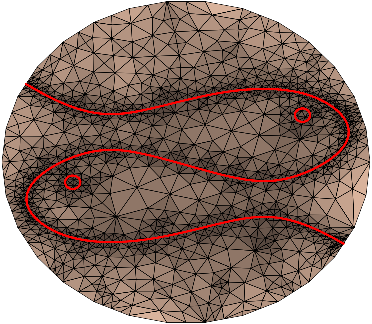

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  - Good AA libraries in C/C++ available

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- We can compute arbitrary formulas on affine forms
  - non-affine operations $\Rightarrow$ minimax approximation

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- IA form $\Rightarrow$ AA form
  - $z \in [a, b] \Rightarrow \hat{z} = z_0 + z_1\varepsilon_1$ where
    $z_0 = (a + b)/2$
    $z_1 = (b - a)/2$

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Replacing ε_1 by $x - 1$:

$r : y = 2x$

$s : y = 2x - 1$

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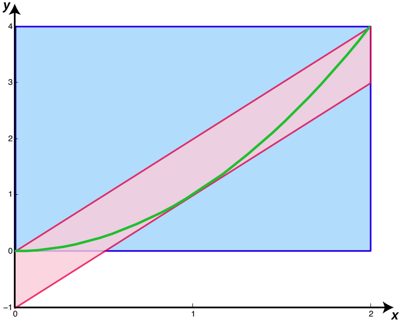
\end{aligned}
$$



Replacing ε_1 by $x - 1$:

$r : y = 2x$

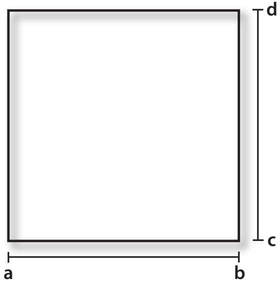
$s : y = 2x - 1$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles:

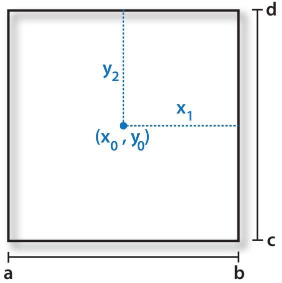
Bounding Implicit Curves with Strips on $\square$

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



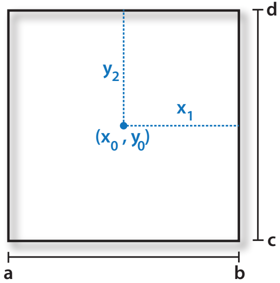
Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\Box)$ with AA

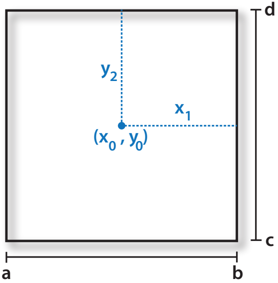


$$\hat{x} = x_0 + x_1\varepsilon_1, \quad x_0 = (a+b)/2, \quad x_1 = (b-a)/2$$

$$\hat{y} = y_0 + y_2\varepsilon_2, \quad y_0 = (c+d)/2, \quad y_2 = (d-c)/2$$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



$$\hat{x} = x_0 + x_1\varepsilon_1, \quad x_0 = (a+b)/2, \quad x_1 = (b-a)/2$$

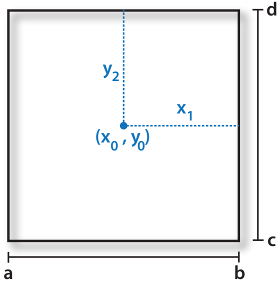
$$\hat{y} = y_0 + y_2\varepsilon_2, \quad y_0 = (c+d)/2, \quad y_2 = (d-c)/2$$

AA form of f

$$\hat{f} = f_0 + f_1\varepsilon_1 + f_2\varepsilon_2 + f_3\varepsilon_3 + \cdots + f_n\varepsilon_n$$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



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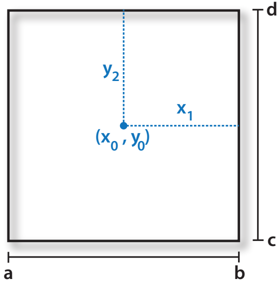
AA form of f

$$\hat{f} = f_0 + f_1\varepsilon_1 + f_2\varepsilon_2 + f_3\varepsilon_3 + \cdots + f_n\varepsilon_n$$

$\varepsilon_3, \ldots, \varepsilon_n$ are noise symbols related to non-affine operations

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



$$\hat{x} = x_0 + x_1\varepsilon_1, \quad x_0 = (a+b)/2, \quad x_1 = (b-a)/2$$

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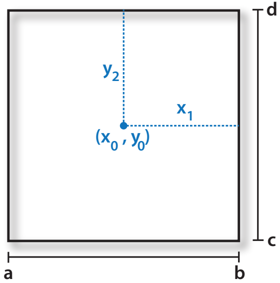
AA form of f

$$\hat{f} = f_0 + f_1\varepsilon_1 + f_2\varepsilon_2 + f_3\varepsilon_3 + \cdots + f_n\varepsilon_n$$

higher-order terms can be condensed $\Rightarrow$ $f_3 = |f_3| + \cdots + |f_n|$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



$$\hat{x} = x_0 + x_1\varepsilon_1, \quad x_0 = (a+b)/2, \quad x_1 = (b-a)/2$$

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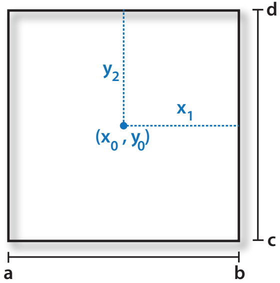
AA form of f

$$\hat{f} = f_0 + f_1\varepsilon_1 + f_2\varepsilon_2 + f_3\varepsilon_3$$

higher-order terms can be condensed $\Rightarrow f_3 = |f_3| + \cdots + |f_n|$

Bounding Implicit Curves with Strips on $\square$

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



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AA form of f

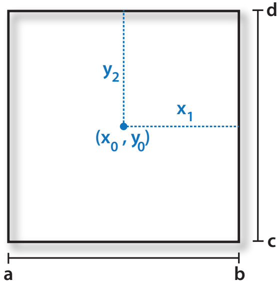
$$\hat{f} = f_0 + f_1\varepsilon_1 + f_2\varepsilon_2 + f_3\varepsilon_3$$

Spatial criteria

$$0 \notin [\hat{f}(\square)] \Rightarrow \texttt{discard}(\square)$$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\Box)$ with AA



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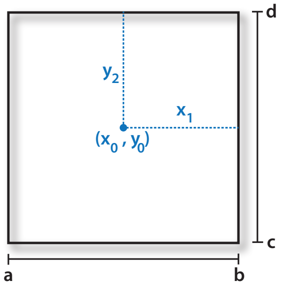
AA form of f

$$\hat{f} = f_0 + f_1\varepsilon_1 + f_2\varepsilon_2 + f_3\varepsilon_3$$

Geometric bounds using the AA form of $\hat{f}$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



$$\hat{x} = x_0 + x_1\varepsilon_1, \quad x_0 = (a+b)/2, \quad x_1 = (b-a)/2$$

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AA form of f

$$\hat{f} = f_0 + f_1\varepsilon_1 + f_2\varepsilon_2 + f_3\varepsilon_3$$

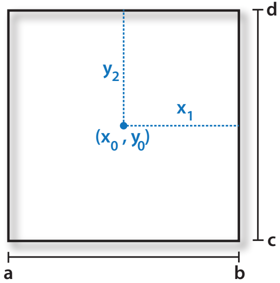
Geometric bounds using the AA form of $\hat{f}$

the graph of $z = f(x, y)$ over □ is sandwiched between the planes:

$$z = f_0 + f_1\varepsilon_1 + f_2\varepsilon_2 \pm f_3$$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



$$\varepsilon_1 = \frac{x - x_0}{x_1} \qquad \varepsilon_2 = \frac{y - y_0}{y_2}$$

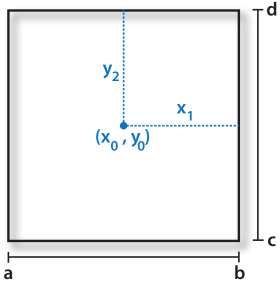
Geometric bounds using the AA form of $\hat{f}$

the graph of $z = f(x, y)$ over □ is sandwiched between the planes:

$$z = f_0 + f_1 \varepsilon_1 + f_2 \varepsilon_2 \pm f_3$$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



$$\varepsilon_1 = \frac{x - x_0}{x_1} \qquad \varepsilon_2 = \frac{y - y_0}{y_2}$$

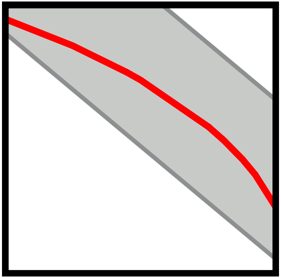
Geometric bounds using the AA form of $\hat{f}$

z in cartesian coordinates:

$$z = f_0 + \frac{f_1}{x_1}(x - x_0) + \frac{f_2}{y_2}(y - y_0) \pm f_3$$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



$$\varepsilon_1 = \frac{x - x_0}{x_1} \qquad \varepsilon_2 = \frac{y - y_0}{y_2}$$

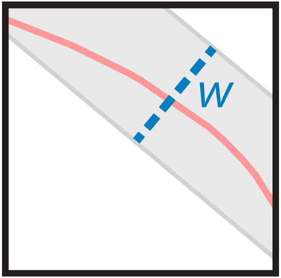
Geometric bounds using the AA form of $\hat{f}$

f is zero inside the strip defined by the two parallel lines:

$$0 = f_0 + \frac{f_1}{x_1}(x - x_0) + \frac{f_2}{y_2}(y - y_0) \pm f_3$$

Bounding Implicit Curves with Strips on □

On axis-aligned rectangles: we need to evaluate $f(\square)$ with AA



The width between the lines

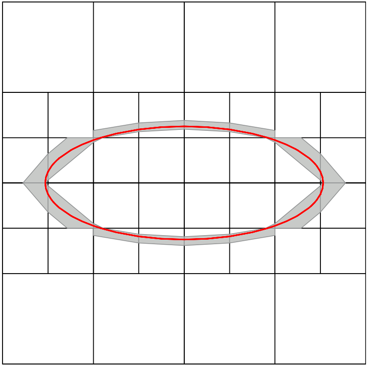
$$w = \frac{2f_3}{\sqrt{\left(\frac{f_1}{x_1}\right)^2 + \left(\frac{f_2}{y_2}\right)^2}}$$

Geometric bounds using the AA form of $\hat{f}$

f is zero inside the strip defined by the two parallel lines:

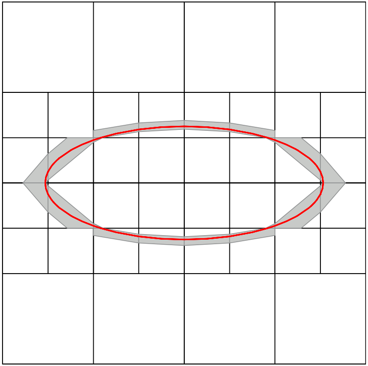
$$0 = f_0 + \frac{f_1}{x_1}(x - x_0) + \frac{f_2}{y_2}(y - y_0) \pm f_3$$

Bounding Implicit Curves with Strips on □



$$\frac{x^2}{6} + y^2 = 1$$

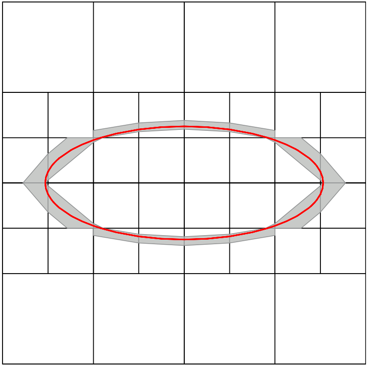
Bounding Implicit Curves with Strips on □



$$\frac{x^2}{6} + y^2 = 1$$

wide strips $\Rightarrow$ high curvature

Bounding Implicit Curves with Strips on □



$$\frac{x^2}{6} + y^2 = 1$$

wide strips $\Rightarrow$ high curvature

Geometric criteria

$w > \textit{threshold} \Rightarrow$ `subdivide(□)`

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation
- adaptive quadtree

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

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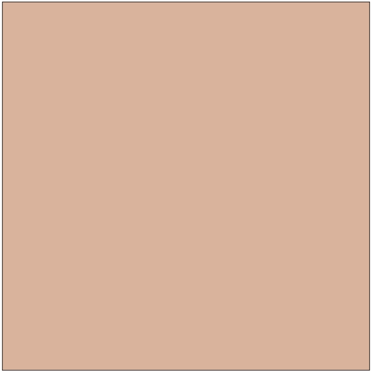
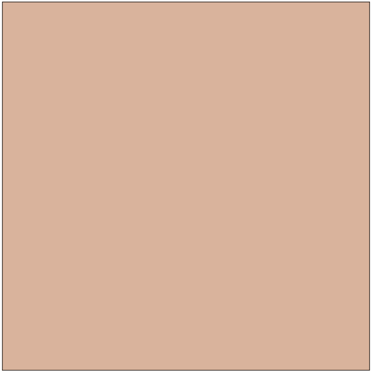


IA

level 0

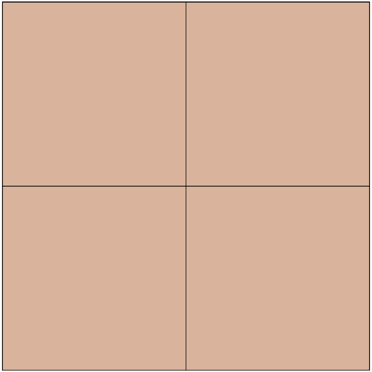


AA

Bounding Implicit Curves with Strips on □

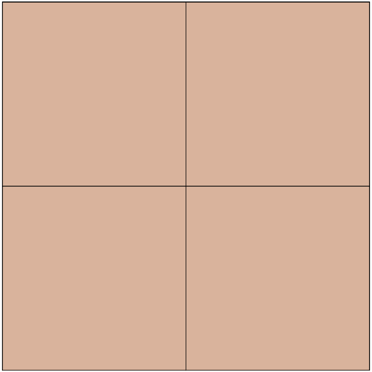
Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation
- adaptive quadtree



IA

level 1



AA

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation
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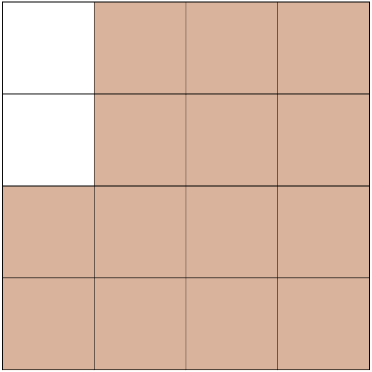
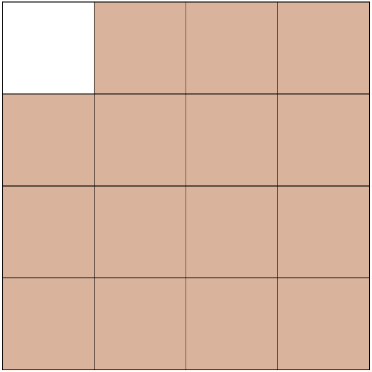


IA

level 2



AA

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

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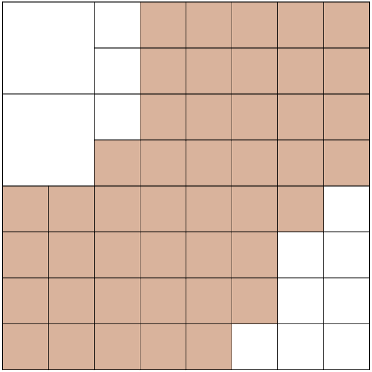
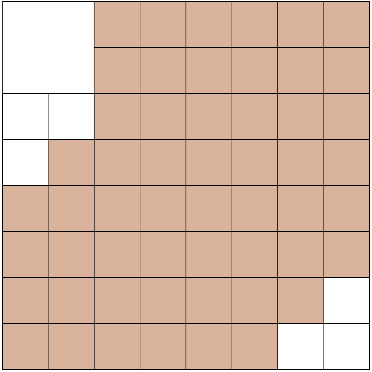


level 3



Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

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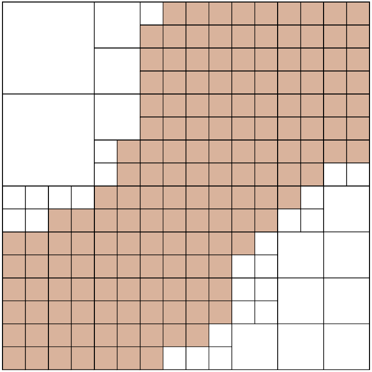
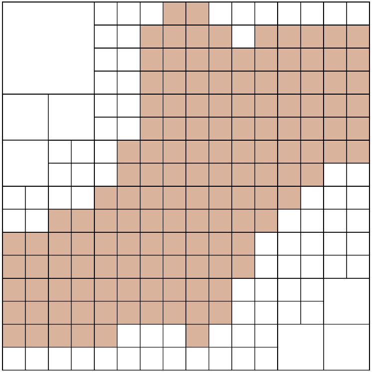


IA

level 4



AA

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

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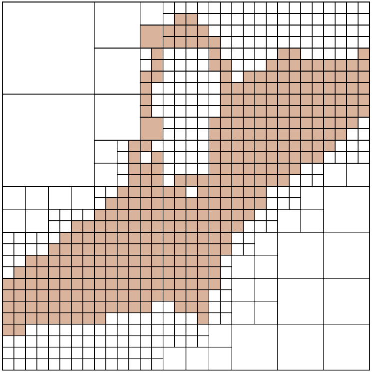
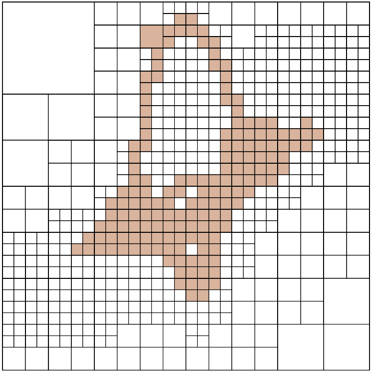
- adaptive quadtree



IA

level 5



AA

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

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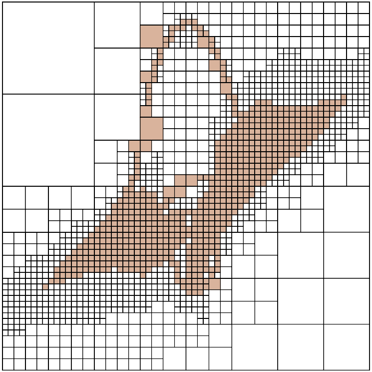
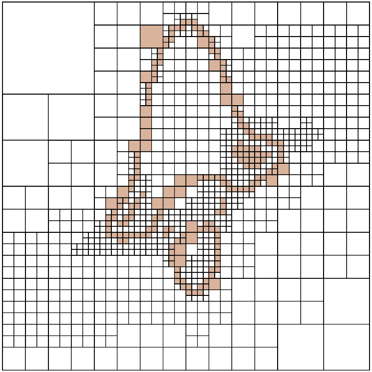


IA

level 6

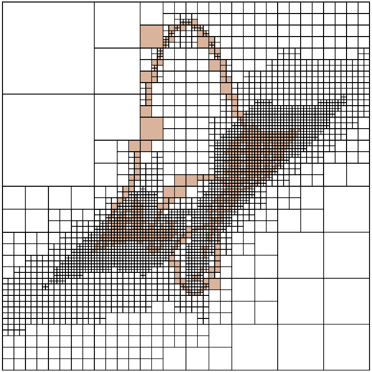


AA

Bounding Implicit Curves with Strips on □

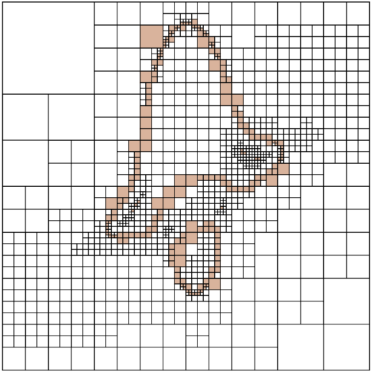
Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation
- adaptive quadtree



IA

level 7

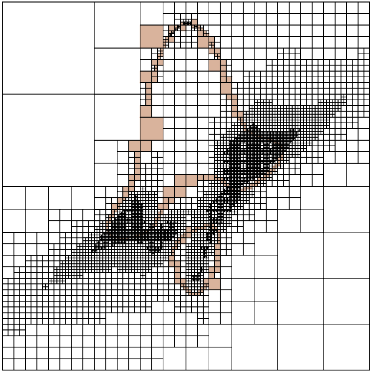


AA

Bounding Implicit Curves with Strips on □

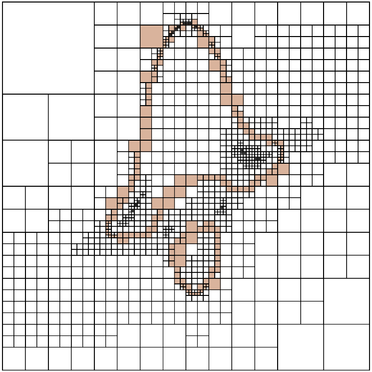
Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation
- adaptive quadtree



IA

level 8



AA

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation
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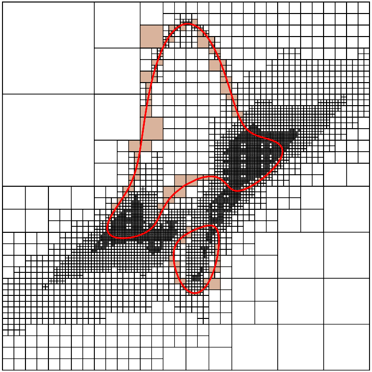
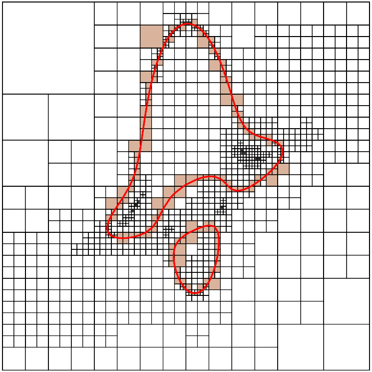


IA

level 8



AA

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

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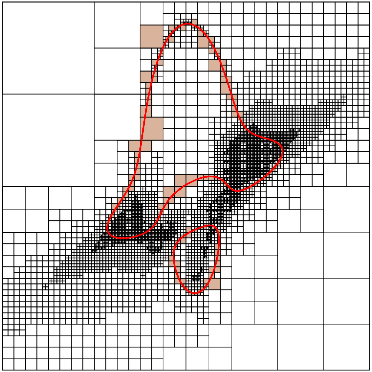
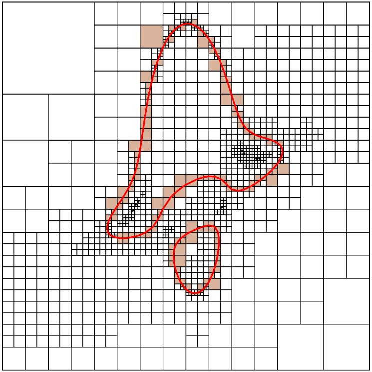


IA

#cells visited: 6997

level 8



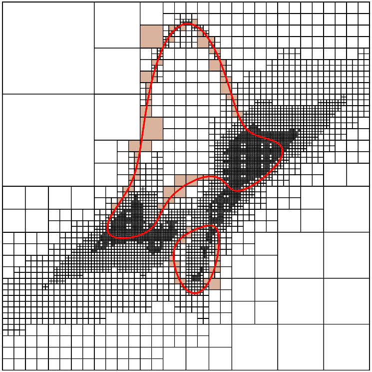
AA

#cells visited: 1697

Bounding Implicit Curves with Strips on □

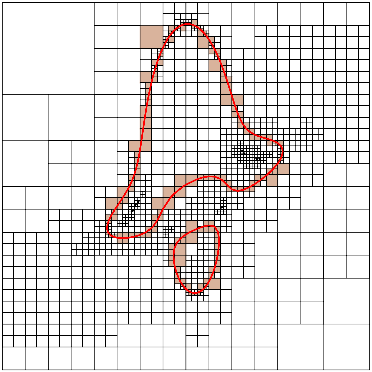
Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation
- adaptive quadtree



IA
#leaves: 341

level 8

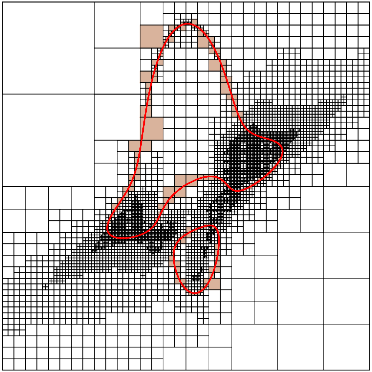


AA
#leaves: 221

Bounding Implicit Curves with Strips on □

Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

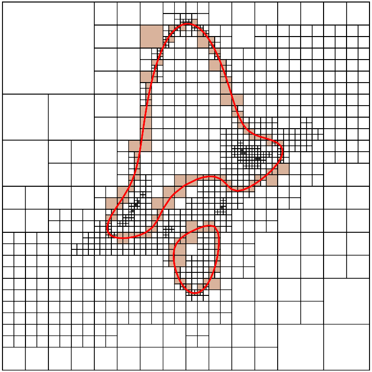
- requires the evaluation ∇f using IA and automatic differentiation
- adaptive quadtree



IA

CPU time: 394 msec

level 8



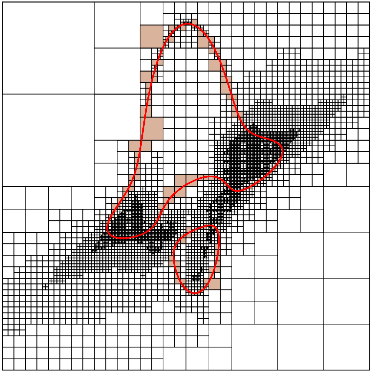
AA

CPU time: 139 msec

Bounding Implicit Curves with Strips on □

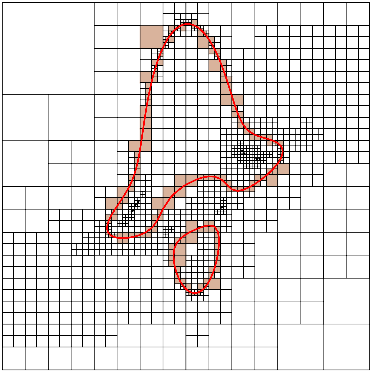
Comparing with IA: method proposed by Lopes *et al.* in SIBGRAPI 2001

- requires the evaluation ∇f using IA and automatic differentiation
- adaptive quadtree



IA
linear convergence

level 8



AA
quadratic convergence

On parallelograms:

Bounding Implicit Curves with Strips on $\Diamond$

On parallelograms:

evaluate $f(\Diamond)$ with AA $\Rightarrow$ write ε_1 and ε_2 in terms of x and y

Bounding Implicit Curves with Strips on $\Diamond$

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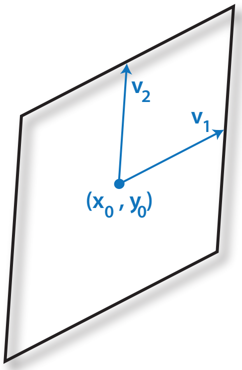

Bounding Implicit Curves with Strips on $\Diamond$

On parallelograms:

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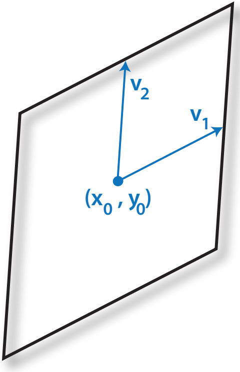
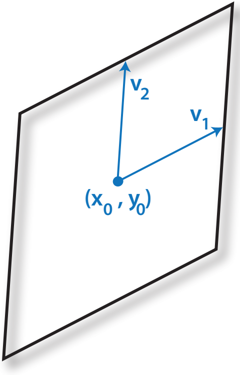


$$v_1 = (x_1, y_1) \qquad v_2 = (x_2, y_2)$$

Bounding Implicit Curves with Strips on $\Diamond$

On parallelograms:

evaluate $f(\Diamond)$ with AA $\Rightarrow$ write ε_1 and ε_2 in terms of x and y



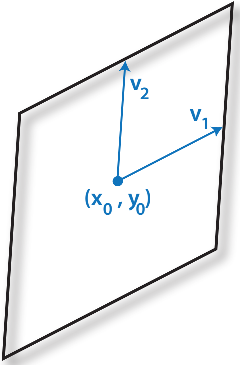
$$v_1 = (x_1, y_1) \qquad v_2 = (x_2, y_2)$$

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 \qquad \hat{y} = y_0 + y_1\varepsilon_1 + y_2\varepsilon_2$$

Bounding Implicit Curves with Strips on $\Diamond$

On parallelograms:

evaluate $f(\Diamond)$ with AA $\Rightarrow$ write ε_1 and ε_2 in terms of x and y



$$v_1 = (x_1, y_1) \qquad v_2 = (x_2, y_2)$$

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 \qquad \hat{y} = y_0 + y_1\varepsilon_1 + y_2\varepsilon_2$$

In matrix form

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

Bounding Implicit Curves with Strips on $\Diamond$

On parallelograms:

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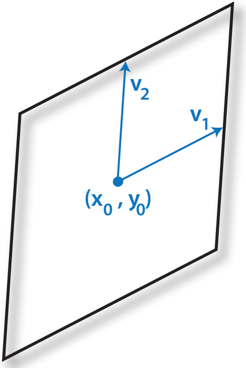

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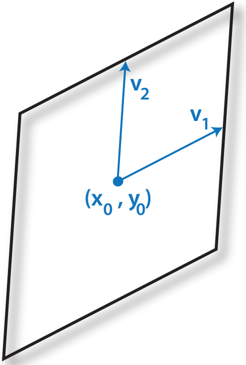
In matrix form

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$$

Bounding Implicit Curves with Strips on $\Diamond$

On parallelograms:

evaluate $f(\Diamond)$ with AA $\Rightarrow$ write ε_1 and ε_2 in terms of x and y



$$v_1 = (x_1, y_1) \qquad v_2 = (x_2, y_2)$$

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In matrix form

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$$

the matrix is invertible $\iff$ the parallelogram is not degenerate

Bounding Implicit Curves with Strips on $\triangle$

On triangles:

Bounding Implicit Curves with Strips on $\triangle$

On triangles: replace the evaluation of $f(\triangle) \Rightarrow f(\Diamond)$ with AA

Bounding Implicit Curves with Strips on $\triangle$

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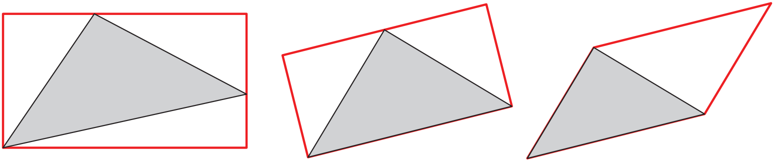
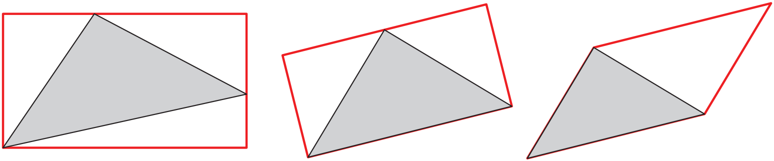


Bounding Implicit Curves with Strips on $\triangle$

On triangles: replace the evaluation of $f(\triangle) \Rightarrow f(\Diamond)$ with AA

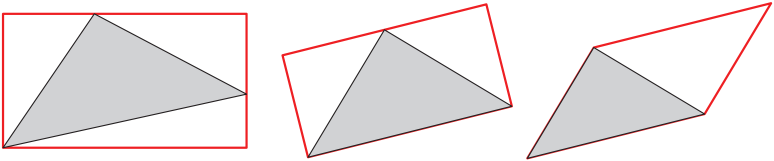
- include a triangle into a parallelogram
  - evaluate f outside of its domain



Bounding Implicit Curves with Strips on $\triangle$

On triangles: replace the evaluation of $f(\triangle) \Rightarrow f(\Diamond)$ with AA

- include a triangle into a parallelogram
  - evaluate f outside of its domain
  - it does not work for surfaces



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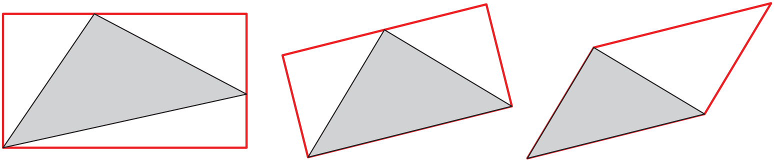

- split a triangle in three parallelograms

Bounding Implicit Curves with Strips on $\triangle$

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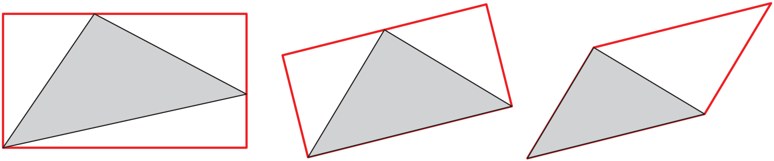

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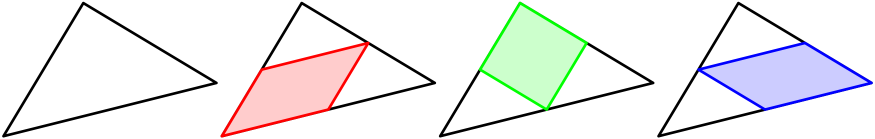
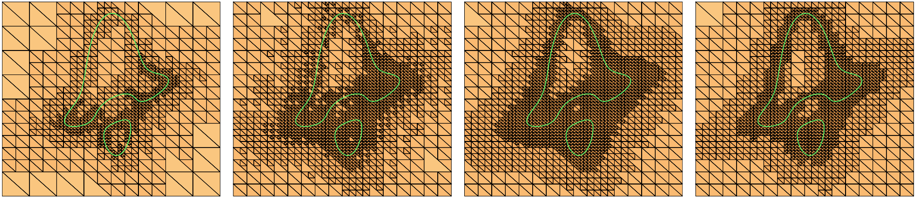


Bounding Implicit Curves with Strips on △



(a) decomposition (b) reflection (c) smallest BB (d) AABB

strategy	time	output	visited	leaves	AA	seg
a	33	**1445**	**1805**	**250**	4604	502
b	**25**	2909	3878	298	3878	**298**
c	28	3392	4522	318	4522	318
d	**25**	2882	3842	316	**3842**	316

Our Adaptive Method

procedure $\mathit{Explore}(\triangle)$

$\Diamond_1, \Diamond_2, \Diamond_3 \leftarrow \mathit{Parallelograms}(\triangle)$

$\hat{f}_i \leftarrow f(\Diamond_i)$ with AA

if $0 \in [\hat{f}_i]$ for some i **then**

$w_i \leftarrow$ width of $\hat{f}$ in $\Diamond_i$

if $w_i \leq \epsilon_{user}$, for all i **then**

$\mathit{Approximate}(\triangle)$

else

$\triangle_i \leftarrow \mathit{Subdivide}(\triangle)$

for each i, $\mathit{Explore}(\triangle_i)$

end

end

end

Our Adaptive Method

procedure *Explore*($\triangle$)
  $\Diamond_1, \Diamond_2, \Diamond_3 \leftarrow \mathit{Parallelograms}(\triangle)$
  $\hat{f}_i \leftarrow f(\Diamond_i)$ with AA
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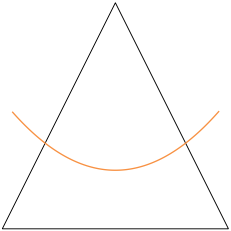
      $\mathit{Approximate}(\triangle)$
    else
      $\triangle_i \leftarrow \mathit{Subdivide}(\triangle)$
      for each i, $\mathit{Explore}(\triangle_i)$
    end
  end
end



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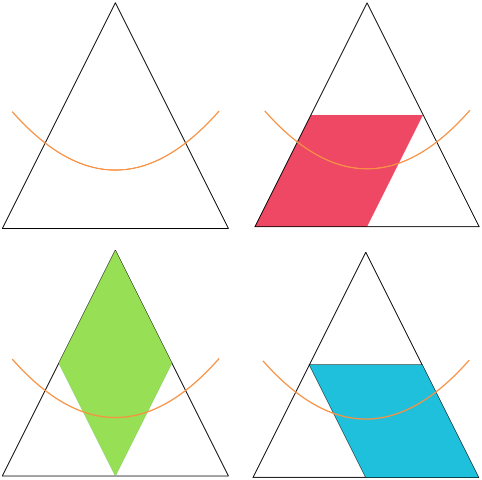
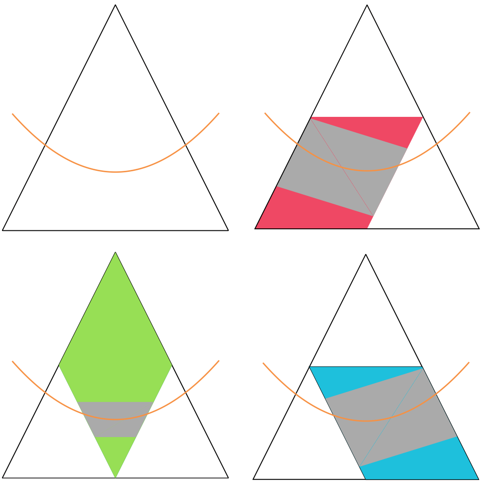
        if $w_i \leq \epsilon_{user}$, for all i **then**
            Approximate($\triangle$)
        else
            $\triangle_i \leftarrow$ *Subdivide*($\triangle$)
            for each i, *Explore*($\triangle_i$)
        end
    end
end



Our Adaptive Method

procedure *Explore*$(\triangle)$
  $\lozenge_1, \lozenge_2, \lozenge_3 \leftarrow$ *Parallelograms*$(\triangle)$
  $\hat{f}_i \leftarrow f(\lozenge_i)$ with AA
  if $0 \in [\hat{f}_i]$ for some i **then**
    $w_i \leftarrow$ width of $\hat{f}$ in $\lozenge_i$
    if $w_i \leq \epsilon_{user}$, for all i **then**
      Approximate$(\triangle)$
    else
      $\triangle_i \leftarrow$ *Subdivide*$(\triangle)$
      for each i, *Explore*$(\triangle_i)$
    end
  end
end



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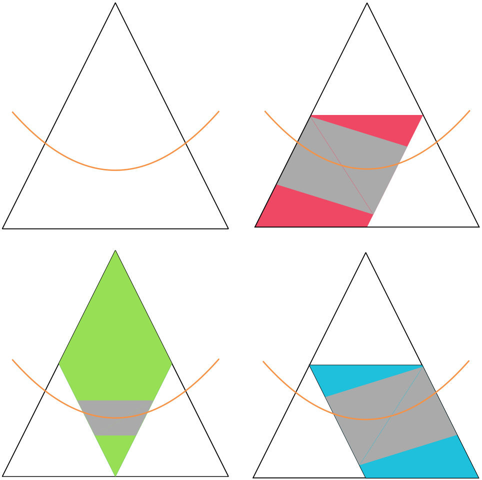
    end
  end
end



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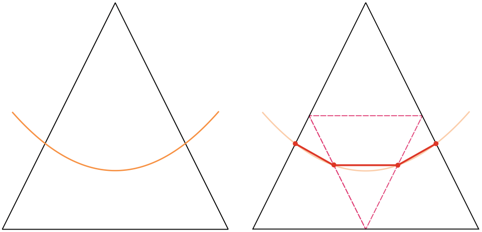

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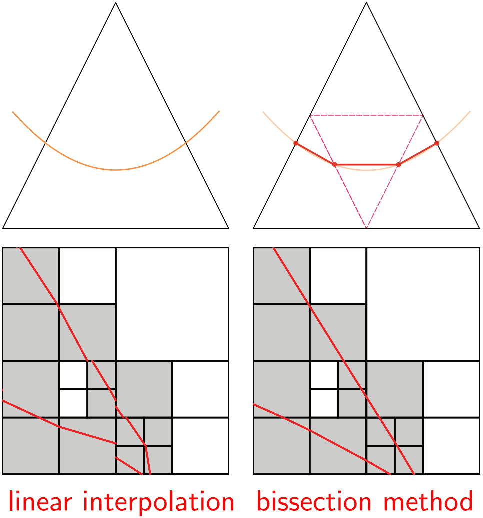

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    else

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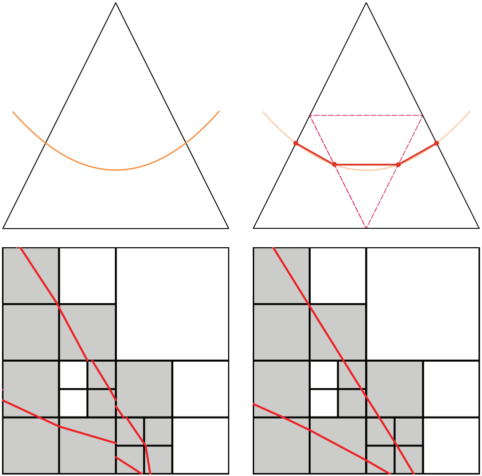
      for each i, *Explore* $(\triangle_i)$

    end

  end

end



linear interpolation   bissection method

Our Adaptive Method

Our method does not care what mesh subdivision method is used

Our Adaptive Method

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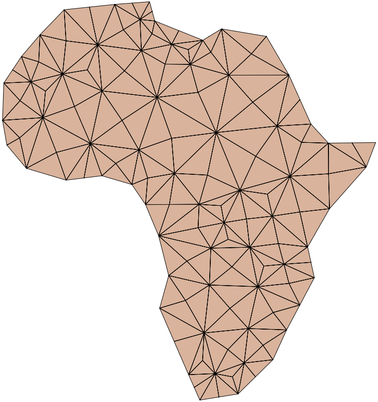
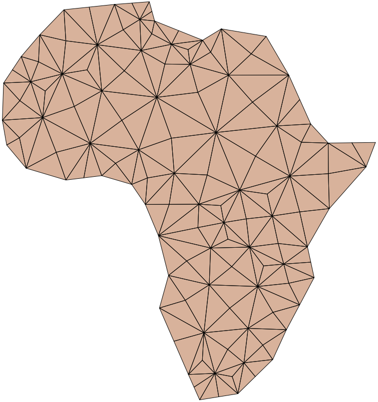


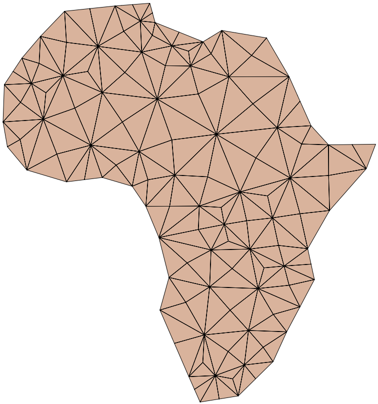
triangle soup



mesh with connecticity

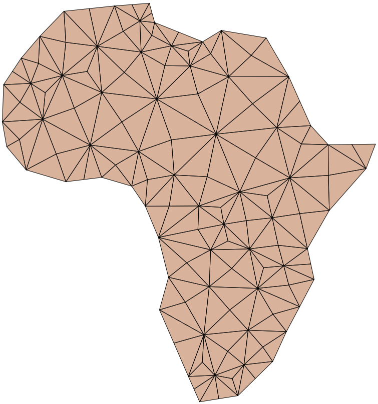
Our Adaptive Method

Our method does not care what mesh subdivision method is used



triangle soup

midpoint splitting



mesh with connecticity

$\sqrt{3}$, J_1^a, 4-8 meshes, ...

Our Adaptive Method

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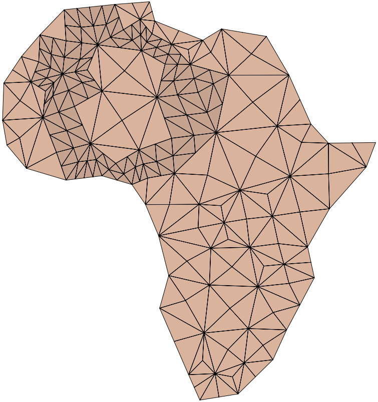
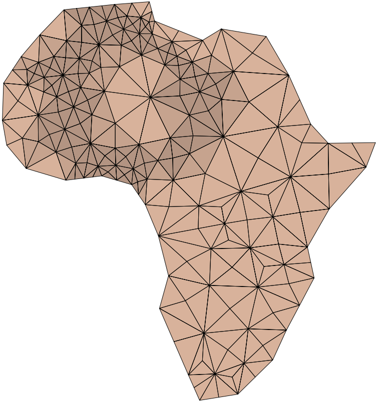


triangle soup

$\#\triangle = 193$



mesh with connecticity

$\#\triangle = 193$

Our Adaptive Method

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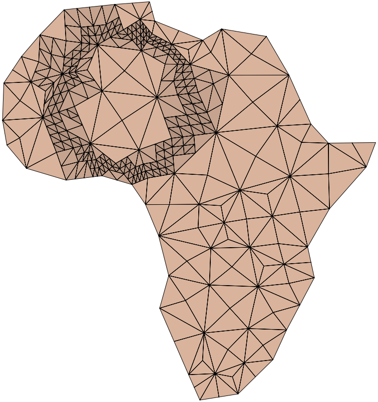
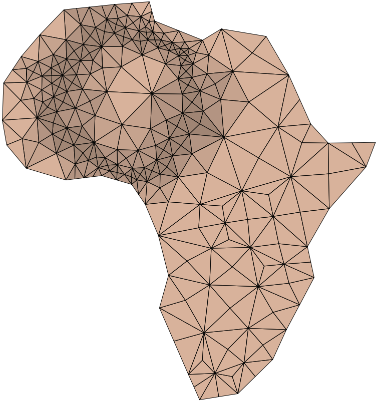


triangle soup

$\# \triangle = 307$

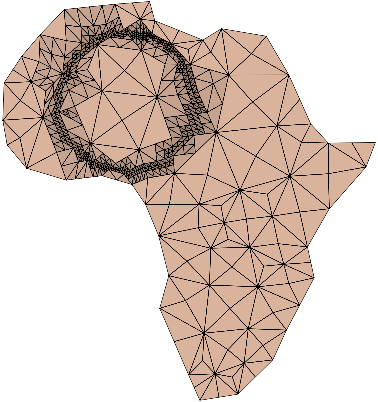


mesh with connecticity

$\# \triangle = 325$

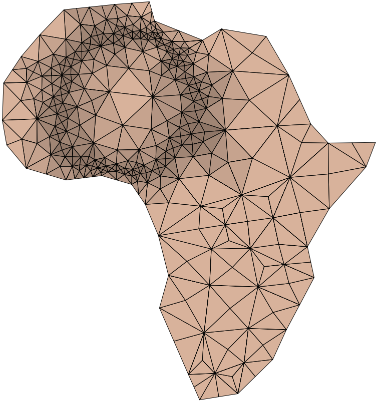
Our Adaptive Method

Our method does not care what mesh subdivision method is used



triangle soup

$\#\triangle = 512$

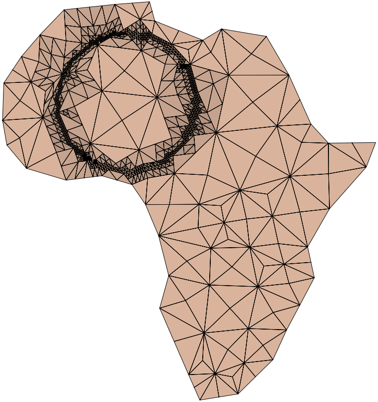


mesh with connecticity

$\#\triangle = 427$

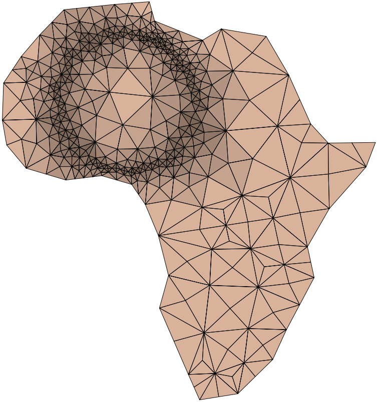
Our Adaptive Method

Our method does not care what mesh subdivision method is used



triangle soup

$\#\triangle = 922$

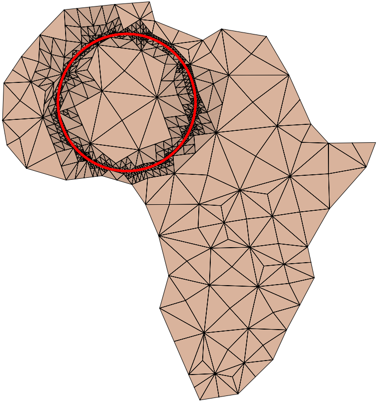


mesh with connecticity

$\#\triangle = 574$

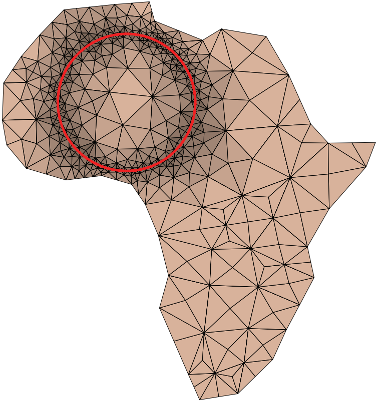
Our Adaptive Method

Our method does not care what mesh subdivision method is used



triangle soup

$\#\triangle = 1384$

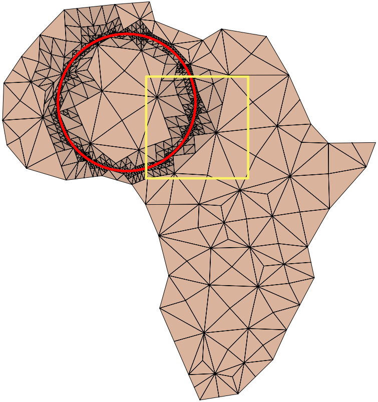


mesh with connecticity

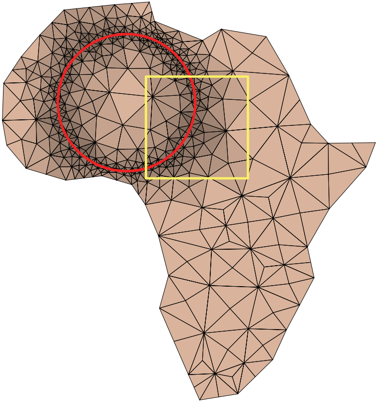
$\#\triangle = 779$

Our Adaptive Method

Our method does not care what mesh subdivision method is used



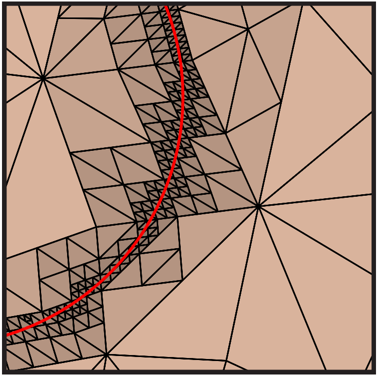
triangle soup
$\#\triangle = 1384$



mesh with connecticity
$\#\triangle = 779$

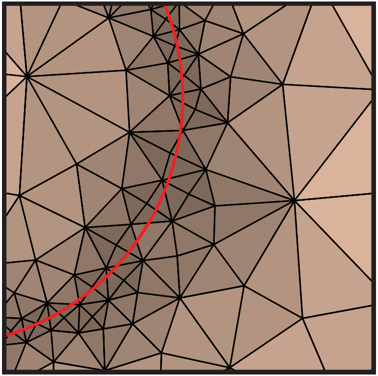
Our Adaptive Method

Our method does not care what mesh subdivision method is used



triangle soup

$\#\triangle = 1384$

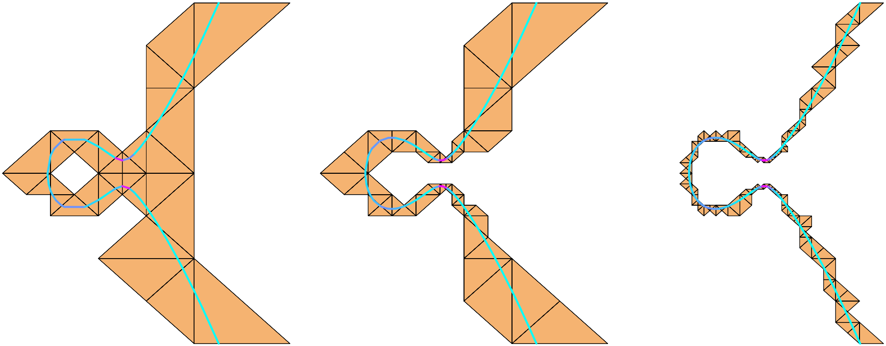


mesh with connecticity

$\#\triangle = 779$

Our Adaptive Method

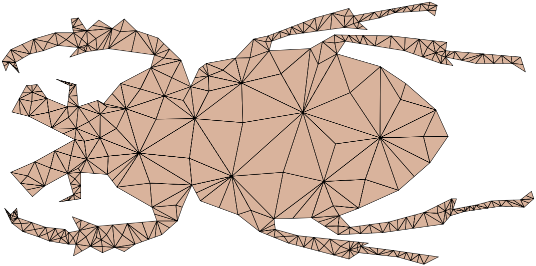
The effect of the geometric criteria on the curve in a triangular quadtree



$\epsilon_{user} = 0.8$ $\epsilon_{user} = 0.4$ $\epsilon_{user} = 0.1$

$$y^2 - x^3 + x = 0.5$$

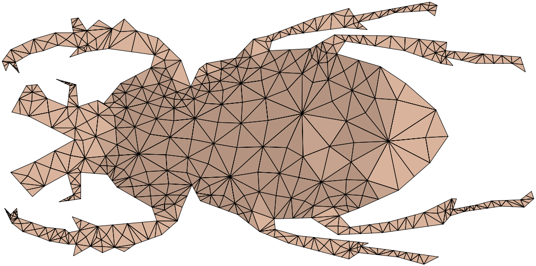
Results



level 0

$$(x+1)^3(1-x)-4y^4=0$$

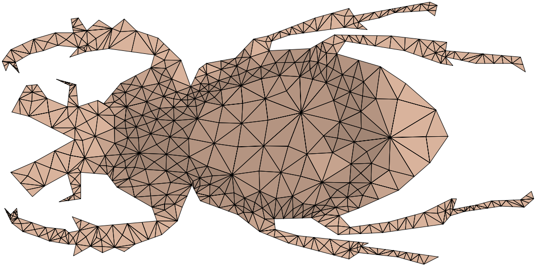
Results



level 1

$$(x+1)^3(1-x) - 4y^4 = 0$$

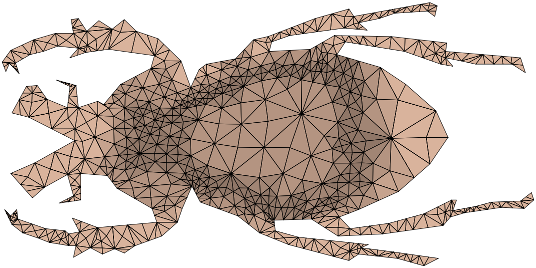
Results



level 2

$$(x+1)^3(1-x)-4y^4=0$$

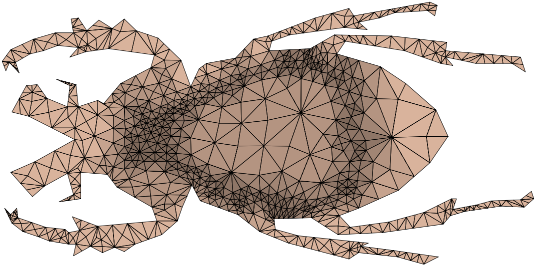
Results



level 3

$$(x+1)^3(1-x)-4y^4=0$$

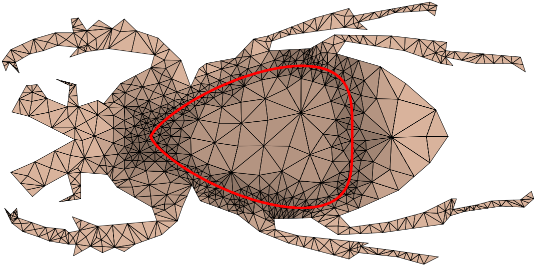
Results



level 4

$$(x+1)^3(1-x)-4y^4=0$$

Results



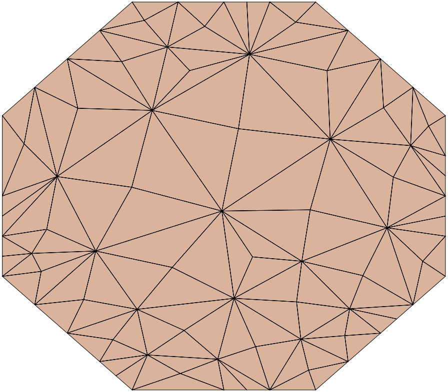
level 4

$\#\triangle_{in} = 940$
$\#\triangle_{out} = 1771$

CPU time = 280 msec

$(x+1)^3(1-x) - 4y^4 = 0$

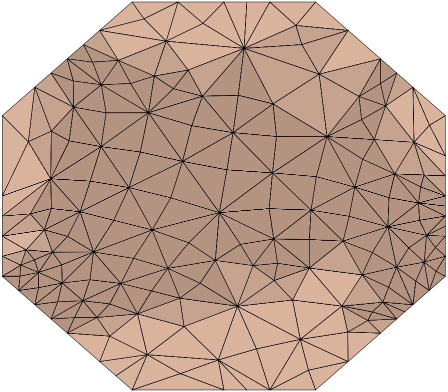
Results



level 0

$$y^2(0.75^2 - x^2) - (x^2 + 1.5y - 0.75^2)^2 = 0$$

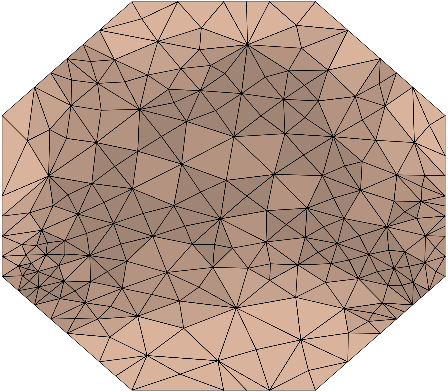
Results



level 1

$$y^2(0.75^2 - x^2) - (x^2 + 1.5y - 0.75^2)^2 = 0$$

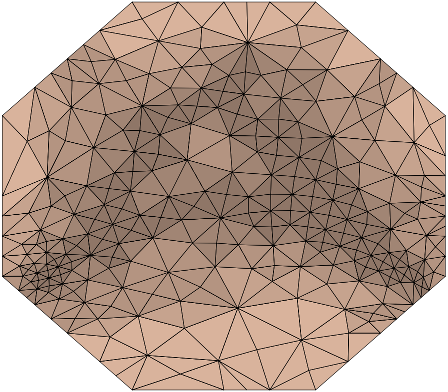
Results



level 2

$$y^2(0.75^2 - x^2) - (x^2 + 1.5y - 0.75^2)^2 = 0$$

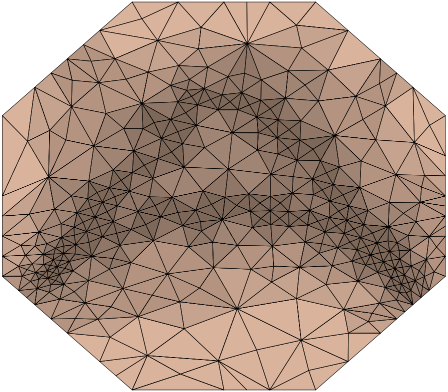
Results



level 3

$$y^2(0.75^2 - x^2) - (x^2 + 1.5y - 0.75^2)^2 = 0$$

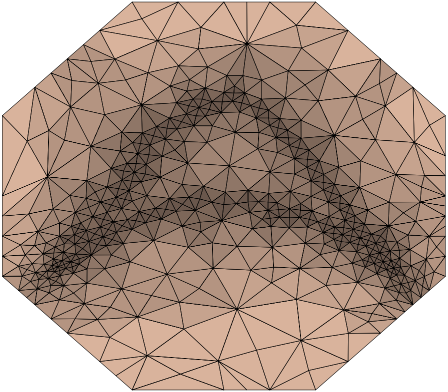
Results



level 4

$$y^2(0.75^2 - x^2) - (x^2 + 1.5y - 0.75^2)^2 = 0$$

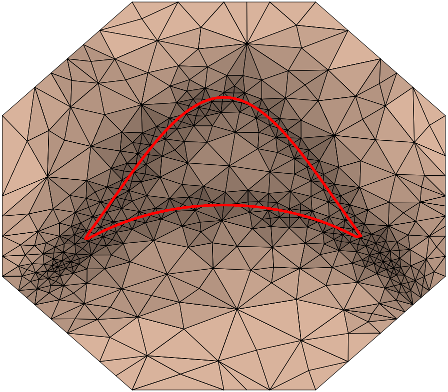
Results



level 5

$$y^2(0.75^2 - x^2) - (x^2 + 1.5y - 0.75^2)^2 = 0$$

Results



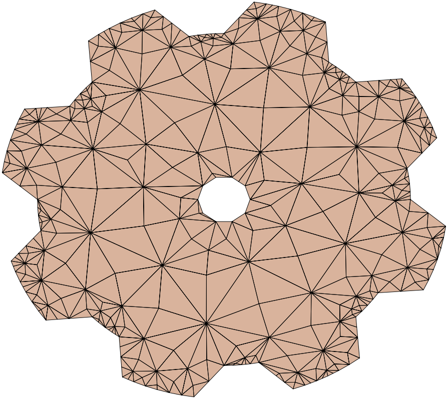
level 5

$\#\triangle_{in} = 126$
$\#\triangle_{out} = 1168$

CPU time = 123 msec

$$y^2(0.75^2 - x^2) - (x^2 + 1.5y - 0.75^2)^2 = 0$$

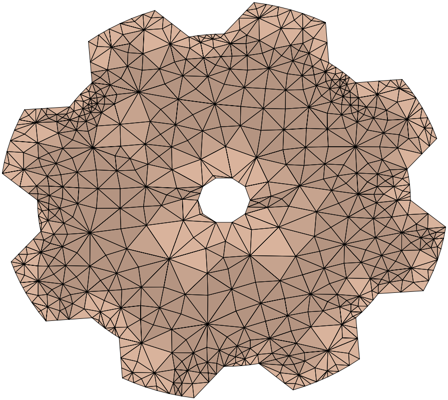
Results



level 0

$$(x^2 + y^2 - 1)^3 - x^2 y^3 = 0$$

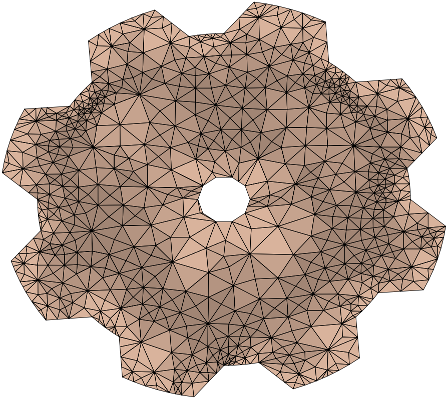
Results



level 1

$$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$$

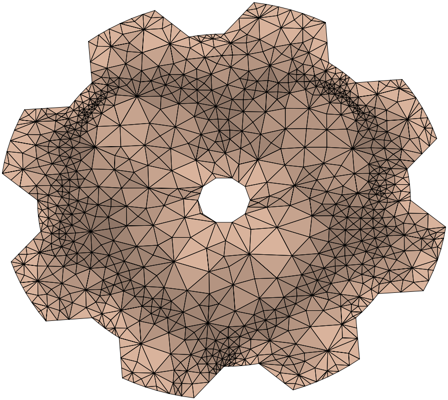
Results



level 2

$$(x^2 + y^2 - 1)^3 - x^2 y^3 = 0$$

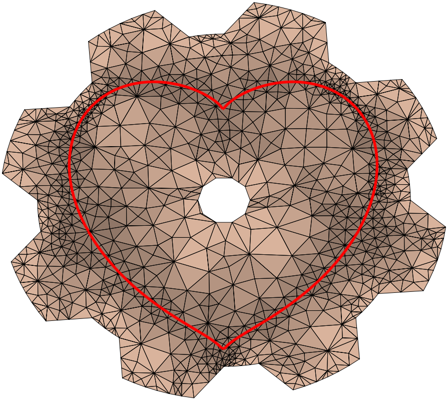
Results



level 3

$$(x^2 + y^2 - 1)^3 - x^2 y^3 = 0$$

Results



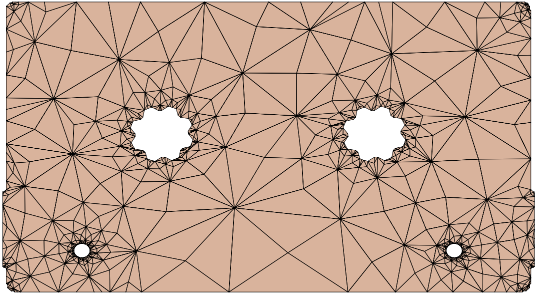
level 3

$\#\triangle_{in} = 1424$
$\#\triangle_{out} = 3298$

CPU time = 547 msec

$$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$$

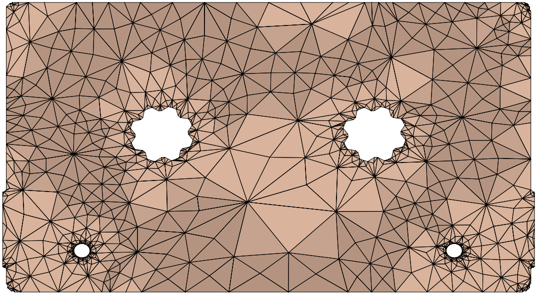
Results



level 0

$$(y - x^2 + 1)^4 + (x^2 + y^2)^4 - 1 = 0$$

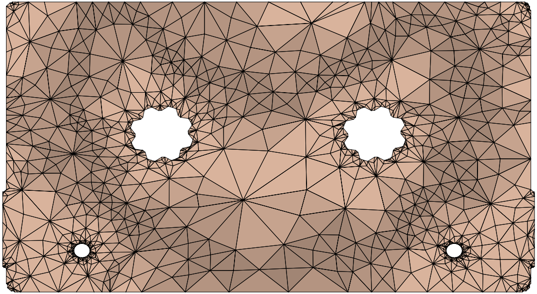
Results



level 1

$$(y - x^2 + 1)^4 + (x^2 + y^2)^4 - 1 = 0$$

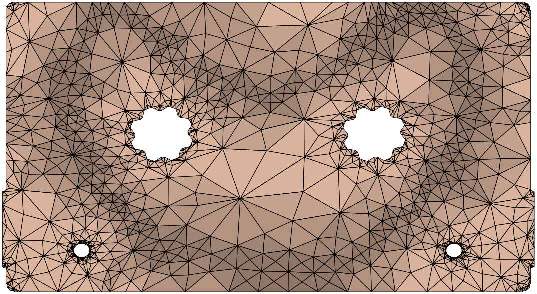
Results



level 2

$$(y - x^2 + 1)^4 + (x^2 + y^2)^4 - 1 = 0$$

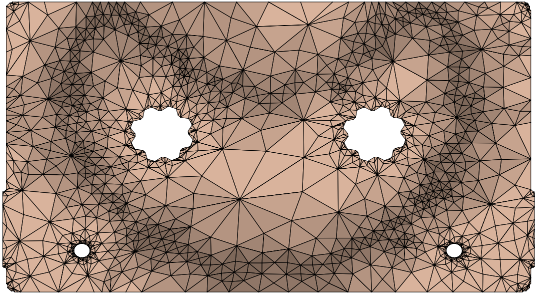
Results



level 3

$$(y - x^2 + 1)^4 + (x^2 + y^2)^4 - 1 = 0$$

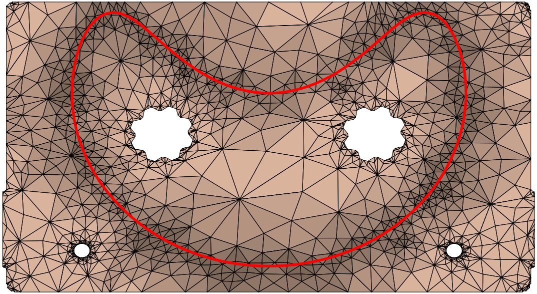
Results



level 4

$$(y - x^2 + 1)^4 + (x^2 + y^2)^4 - 1 = 0$$

Results



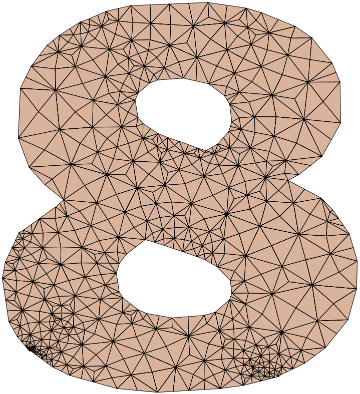
level 4

$\#\triangle_{in} = 1006$
$\#\triangle_{out} = 2134$

CPU time = 391 msec

$$(y - x^2 + 1)^4 + (x^2 + y^2)^4 - 1 = 0$$

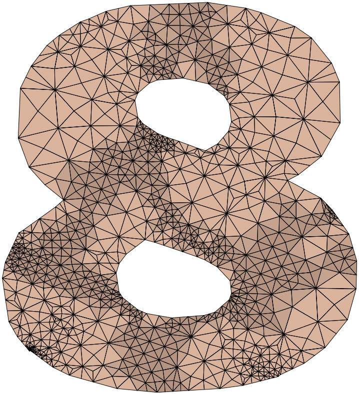
Results



level 0

$$(xy + \cos(x + y))(xy + \sin(x + y)) = 0$$

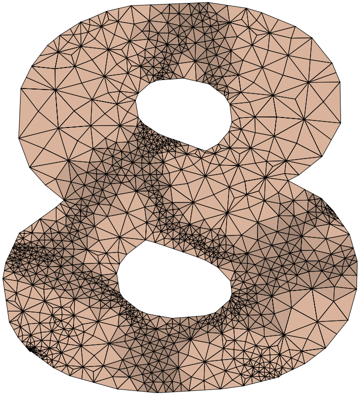
Results



level 1

$$(xy + \cos(x + y))(xy + \sin(x + y)) = 0$$

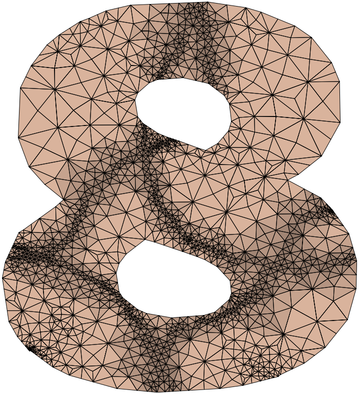
Results



level 2

$$(xy + \cos(x+y))(xy + \sin(x+y)) = 0$$

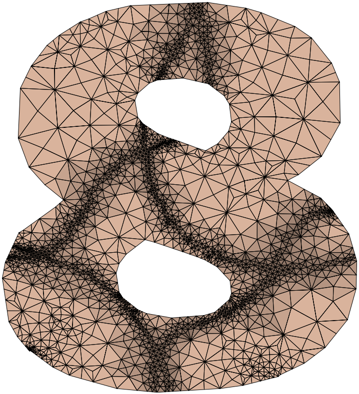
Results



level 3

$$(xy + \cos(x + y))(xy + \sin(x + y)) = 0$$

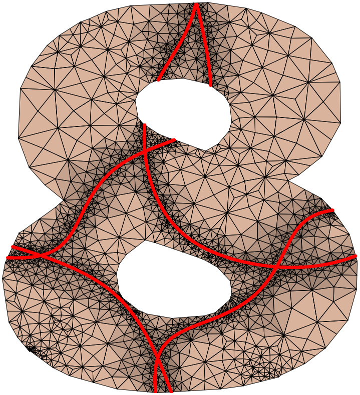
Results



level 4

$$(xy + \cos(x + y))(xy + \sin(x + y)) = 0$$

Results



level 4

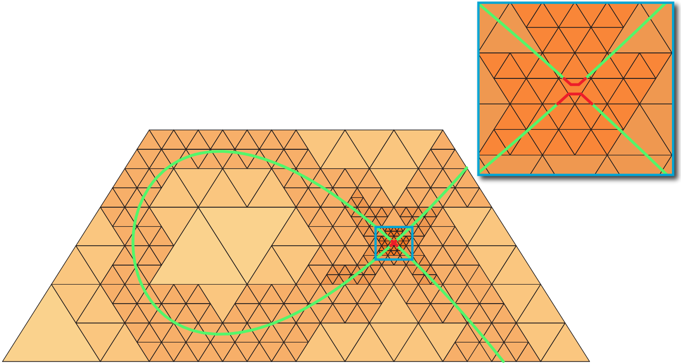
$\#\triangle_{in} = 1032$
$\#\triangle_{out} = 3897$

CPU time = 454 msec

$(xy + \cos(x + y))(xy + \sin(x + y)) = 0$

Results

Our method detects the non-manifold region...



$$y^2 = x^3 + 3x^2$$

...even when the singularity is not recovered!

Bounding Implicit Curves with Strips on Surfaces

On parallelograms in 3D:

Bounding Implicit Curves with Strips on Surfaces

On parallelograms in 3D:

evaluate $f(\Diamond)$ with AA $\Rightarrow$ write ε_1 and ε_2 in terms of x, y and z

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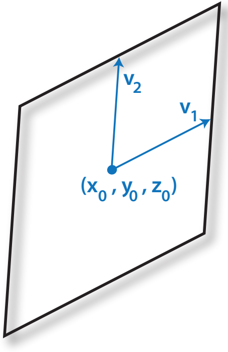

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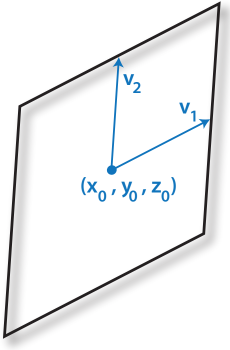

$$v_1 = (x_1, y_1, z_1) \qquad v_2 = (x_2, y_2, z_2)$$

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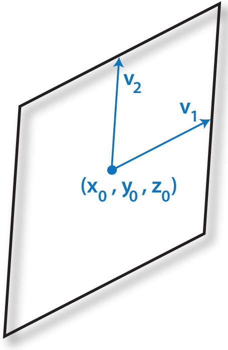

$$v_1 = (x_1, y_1, z_1) \qquad v_2 = (x_2, y_2, z_2)$$

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2$$
$$\hat{y} = y_0 + y_1\varepsilon_1 + y_2\varepsilon_2$$
$$\hat{z} = z_0 + z_1\varepsilon_1 + z_2\varepsilon_2$$

Bounding Implicit Curves with Strips on Surfaces

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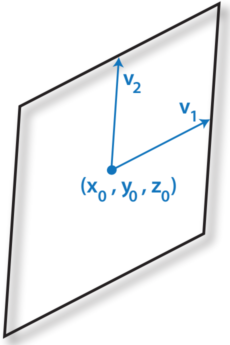

$$v_1 = (x_1, y_1, z_1) \qquad v_2 = (x_2, y_2, z_2)$$

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2$$
$$\hat{y} = y_0 + y_1\varepsilon_1 + y_2\varepsilon_2$$
$$\hat{z} = z_0 + z_1\varepsilon_1 + z_2\varepsilon_2$$

In matrix form

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ z_1 & z_2 \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

Bounding Implicit Curves with Strips on Surfaces

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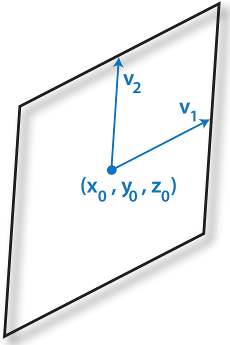

$$v_1 = (x_1, y_1, z_1) \qquad v_2 = (x_2, y_2, z_2)$$

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$$\hat{y} = y_0 + y_1\varepsilon_1 + y_2\varepsilon_2$$

$$\hat{z} = z_0 + z_1\varepsilon_1 + z_2\varepsilon_2$$

In matrix form

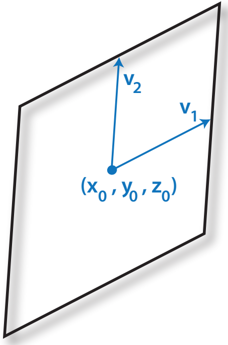
$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ z_1 & z_2 \end{bmatrix}^+ \cdot \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

$\mathbf{B}^+ = (\mathbf{B}^\top \mathbf{B})^{-1}\mathbf{B}^\top$ is the pseudoinverse of a matrix $\mathbf{B}$

Bounding Implicit Curves with Strips on Surfaces

On parallelograms in 3D:

evaluate $f(\Diamond)$ with AA $\Rightarrow$ write ε_1 and ε_2 in terms of x, y and z



$$v_1 = (x_1, y_1, z_1) \qquad v_2 = (x_2, y_2, z_2)$$

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2$$
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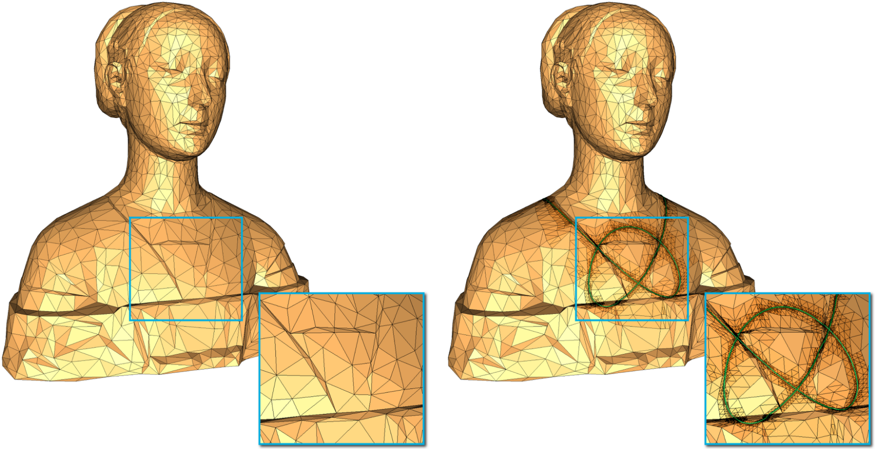
$$\hat{z} = z_0 + z_1\varepsilon_1 + z_2\varepsilon_2$$

In matrix form

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ z_1 & z_2 \end{bmatrix}^+ \cdot \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

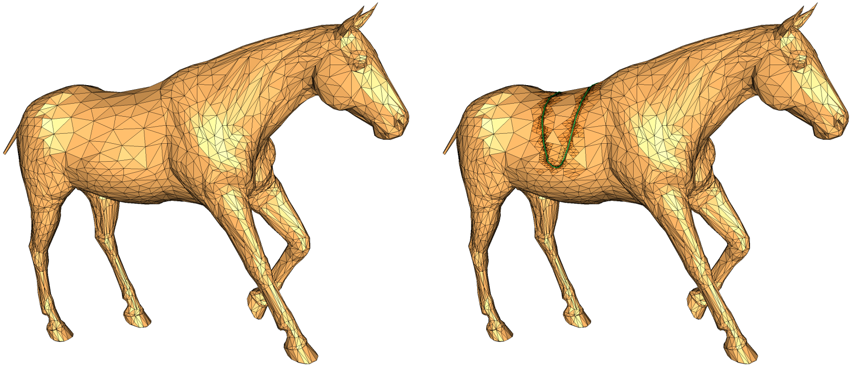
the matrix has full rank $\Longleftrightarrow$ the parallelogram is not degenerate

Results



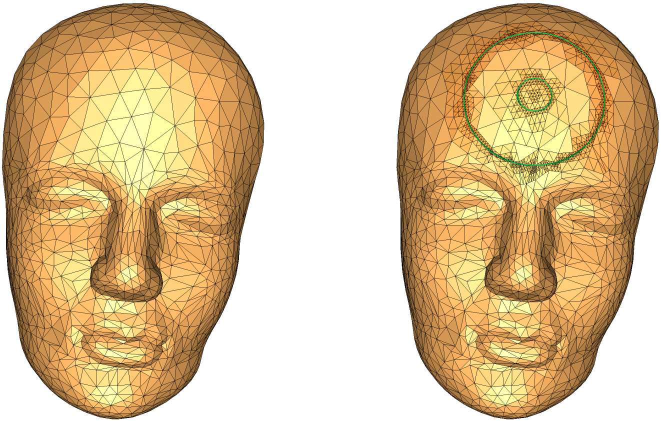
$$y^2(3+2y)-(x^2-1)^2=0$$

Results



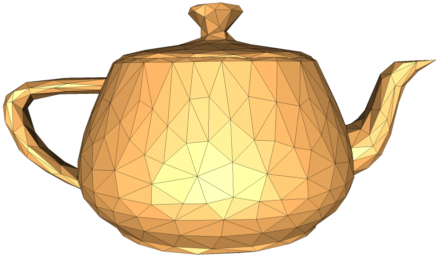
$$x^2 - 48y^2 = 8z = 0$$

Results



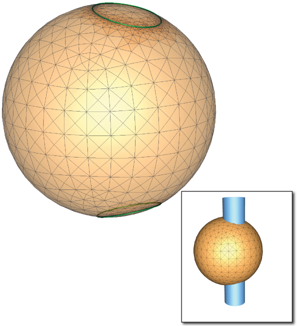
$$xy^2(1 - \sqrt{xy^2}) = 0.04$$

Results

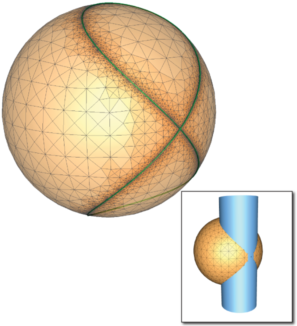


$$(xy-2)(x^2+y^2-1)=0$$

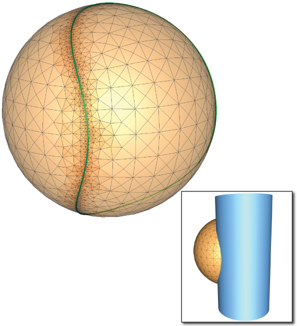
Results



$a = 0.3$



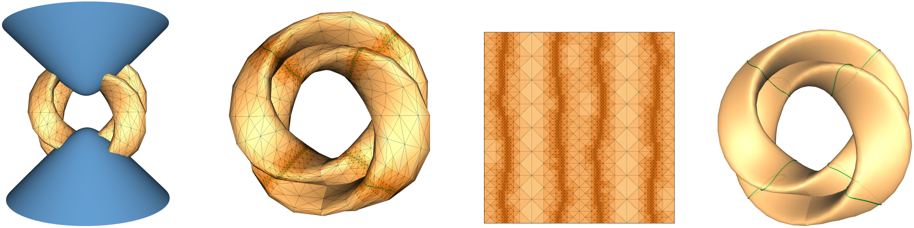
$a = 0.5$



$a = 0.7$

$$(x - a)^2 + y^2 = a^2$$

Results: *implicit* × *parametric*



hyperboloid given implicitly by $x^2 - y^2 - z^2 = 1$

Klein bottle given parametrically by

$$x(u,v) = (2.7 + \cos(u)\sin(v) - \sin(u)\sin(2v))\cos(u),$$
$$y(u,v) = (2.7 + \cos(u)\sin(v) - \sin(u)\sin(2v))\sin(u),$$
$$z(u,v) = \sin(u)\sin(v) + \cos(u)\sin(2v),$$

Approximating Implicit Curves on Plane and Surface Triangulations with Affine Arithmetic

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apneto @ icmc.usp.br

Questions? Thanks!