IMPA – 2011
Aprendizagem Estatística
1ª Lista de Exercícios
Para 25/4

1. MacKay, exercício 2.18.

▷ Exercise 2.18.[2, p.42] Let x and y be dependent random variables with x a binary variable taking values in $\mathcal{A}_X = \{0, 1\}$. Use Bayes' theorem to show that the log posterior probability ratio for x given y is

$$\log \frac{P(x=1 \mid y)}{P(x=0 \mid y)} = \log \frac{P(y \mid x=1)}{P(y \mid x=0)} + \log \frac{P(x=1)}{P(x=0)}. \qquad (2.51)$$

2. MacKay, exercício 2.37

▷ Exercise 2.37.[2] The inhabitants of an island tell the truth one third of the time. They lie with probability 2/3.

On an occasion, after one of them made a statement, you ask another 'was that statement true?' and he says 'yes'.

What is the probability that the statement was indeed true?

3. MacKay, exercício 3.1

▷ Exercise 3.1.[2, p.59] A die is selected at random from two twenty-faced dice on which the symbols 1–10 are written with nonuniform frequency as follows.

Symbol	1	2	3	4	5	6	7	8	9	10
Number of faces of die A	6	4	3	2	1	1	1	1	1	0
Number of faces of die B	3	3	2	2	2	2	2	2	1	1

The randomly chosen die is rolled 7 times, with the following outcomes:

5, 3, 9, 3, 8, 4, 7.

What is the probability that the die is die A?

4. MacKay, exercício 3.3

Exercise 3.3.[3, p.48] Inferring a decay constant

Unstable particles are emitted from a source and decay at a distance x, a real number that has an exponential probability distribution with characteristic length λ. Decay events can only be observed if they occur in a window extending from $x = 1\,\text{cm}$ to $x = 20\,\text{cm}$. N decays are observed at locations $\{x_1, \ldots, x_N\}$. What is λ?

5. Reproduza o exemplo das figuras 2.1-2.3 do livro. Isto é, inicialmente gere 10 médias com distribuição normal de média (0, 1) e variância **I** e 10 médias com distribuição normal de média (1, 0) e variância **I**. A seguir, gere 100 pontos em cada uma das classes, selecionando, para cada ponto, uma das 10 médias ao acaso e gerando um ponto com distribuição normal com centro nesta média e variância **I**/5. Plote os pontos das duas classes utilizando cores ou símbolos diferentes.

6. Utilizando os pontos em http://www.impa.br/~pcezar/cursos/ae/medias.txt, resolva o exercício 2.2 de Hastie et al. Isto é, encontre a fronteira Bayesiana de separação das classes, caracterizando os pontos $\mathbf{x}$ para os quais $P(\text{classe} = 1 \mid \mathbf{X} = \mathbf{x}) \geq P(\text{classe} = 2 \mid \mathbf{X} = \mathbf{x})$, admitindo que as duas classes têm iguais probabilidades a priori.