# IMPA - 2011 Aprendizagem Estatística 1<sup>a</sup> Lista de Exercícios Para 25/4

#### 1. MacKay, exercício 2.18.

 $\triangleright$  Exercise 2.18. [2, p.42] Let x and y be dependent random variables with x a binary variable taking values in  $A_X = \{0,1\}$ . Use Bayes' theorem to show that the log posterior probability ratio for x given y is

$$\log \frac{P(x=1\,|\,y)}{P(x=0\,|\,y)} = \log \frac{P(y\,|\,x=1)}{P(y\,|\,x=0)} + \log \frac{P(x=1)}{P(x=0)}. \tag{2.51}$$

### 2. MacKay, exercício 2.37

▷ Exercise 2.37. [2] The inhabitants of an island tell the truth one third of the time. They lie with probability 2/3.

> On an occasion, after one of them made a statement, you ask another 'was that statement true?' and he says 'yes'.

What is the probability that the statement was indeed true?

## 3. MacKay, exercício 3.1

Exercise 3.1.<sup>[2, p.59]</sup> A die is selected at random from two twenty-faced dice on which the symbols 1-10 are written with nonuniform frequency as follows.

Symbol	1	2	3	4	5	6	7	8	9	10
Number of faces of die A	6	4	3	2	1	1	1	1	1	0
Number of faces of die B	3	3	2	2	2	2	2	2	1	1

The randomly chosen die is rolled 7 times, with the following outcomes:

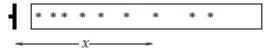
What is the probability that the die is die A?

#### 4. MacKay, exercício 3.3



Exercise 3.3.  $^{[3, p.48]}$  Inferring a decay constant

Unstable particles are emitted from a source and decay at a distance x, a real number that has an exponential probability distribution with characteristic length  $\lambda$ . Decay events can only be observed if they occur in a window extending from  $x = 1 \,\mathrm{cm}$  to  $x = 20 \,\mathrm{cm}$ . N decays are observed at locations  $\{x_1, ..., x_N\}$ . What is  $\lambda$ ?



- 5. Reproduza o exemplo das figuras 2.1-2.3 do livro. Isto é, inicialmente gere 10 médias com distribuição normal de média (0, 1) e variância I e 10 médias com distribuição normal de média (1, 0) e variância I. A seguir, gere 100 pontos em cada uma das classes, selecionando, para cada ponto, uma das 10 médias ao acaso e gerando um ponto com distribuição normal com centro nesta média e variância I/5. Plote os pontos das duas classes utilizando cores ou símbolos diferentes.
- 6. Utilizando os pontos em http://www.impa.br/~pcezar/cursos/ae/medias.txt, resolva o exercício 2.2 de Hastie et al. Isto é, encontre a fronteira Bayesiana de separação das classes, caracterizando os pontos x para os quais P(classe = 1 | X = x) ≥ P(classe = 2 | X = x), admitindo que as duas classes têm iguais probabilidades a priori.