

Probabilidade I. 2017.1  
3rd Exercise Sheet.

Date: 7/April/2017

**1.** Let  $X$  be a r.v. with uniform distribution on the set  $D := \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ . Let  $Z := \frac{X}{\|X\|}$ . Prove or disprove the following claim:  $X$  and  $Z$  are independent.

**2.** Let  $\{X_n\}_{n \geq 1}$  a sequence of i.i.d. random variables uniformly distributed on the interval  $[0, 1]$ . Show that

$$\limsup_{n \rightarrow \infty} \frac{X_{2n+1}}{X_{2n}} = \infty \text{ a.s.}$$

**3.** Let  $X_1, X_2, \dots$  be i.i.d. random variables with  $E(X_1) = 0$  and  $\text{Var}(X_1) = \infty$ . Prove that

$$P(\limsup_{n \rightarrow \infty} \{|X_n| \geq \sqrt{n}\}) = 1.$$