Atauúba: The differential equations of moduler forms.

Ref. Livro: Moduler and Automorphic forms & beyond 2021

Quasi-moduler Forms attached to elliptic Curves 2012

Why generalize moduler forms? $x^n+y^n=1$ Δ Wiles

$\Updownarrow$ 5

E/Q é moduler $\longleftrightarrow$ $f: \mathbb{H} \to \mathbb{C}$

Teoria das Cordas /

Elliptic integrals. $\displaystyle\int_{t_1}^{t_2} \frac{P(x)\,dx}{\sqrt{4x^3-t_2x-t_3}}$ $t_2, t_3 \in \mathbb{C}, \mathbb{R}$ $t_2^3-27t_3^2 \neq 0$

1800 - 1850

Euler-Jacobi-Riemann

t_1, t_2 das raizes (real) de

$$E \cong \left\{ y^2 = 4x^3 - t_2x - t_3 \right\}_{(x,y)\in\mathbb{C}^2} \simeq$$

$$\int_\delta \frac{P(x)\,dx}{y}$$

$\delta \in H_1(E, \mathbb{Z})$

Gauss-Manin connection:

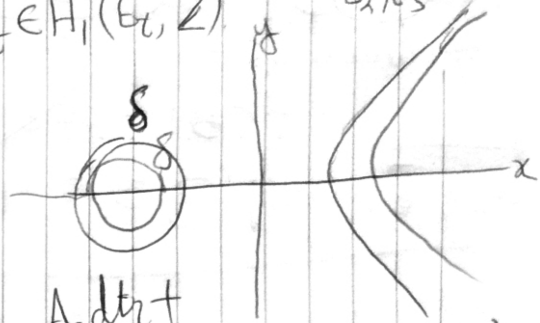
$t=(t_2,t_3)$

funções hol. em t_2, t_3

$\delta_t \in H_1(E_t,\mathbb{Z})$

$$\begin{pmatrix} \frac{\partial}{\partial t_2}\int_\delta \frac{dx}{y} \\ \frac{\partial}{\partial t_2}\int_\delta \frac{x\,dx}{y} \end{pmatrix} = \begin{pmatrix} \times & \cancel{\times} \\ & A_2 & \\ \times & \times \end{pmatrix} \begin{pmatrix} \int_\delta \frac{dx}{y} \\ \int_\delta \frac{x\,dx}{y} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial}{\partial t_3}\int \\ \frac{\partial}{\partial t_3}\int \end{pmatrix} = \begin{pmatrix} A_3 \end{pmatrix} \begin{pmatrix} \\ \end{pmatrix}$$



$A = A_2\,dt_2 + A_3\,dt_3$

$E_{t_2,t_3}(\mathbb{R})$

$\rightarrow$ $\dim H^1_{dR}(E_t) = 2$

$\Delta = 27t_3^2 - t_2^3$

$\alpha = 3t_3 - 2t_2\,dt$

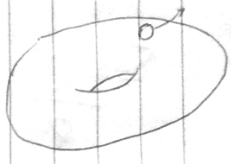
$\nabla : H^1_{dR} \to \Omega^1$

$$\begin{pmatrix} \nabla\left[\frac{dx}{y}\right] \\ \nabla\left[\frac{x\,dx}{y}\right] \end{pmatrix} \longleftrightarrow \begin{pmatrix} d\int \frac{dx}{y} \\ d\int \frac{x\,dx}{y} \end{pmatrix} = \begin{pmatrix} -\frac{1}{12}\frac{d\Delta}{\Delta} & \frac{3}{2}\alpha \\ \frac{1}{8}t_2\alpha & \frac{1}{12}\frac{d\Delta}{\Delta} \end{pmatrix} \begin{pmatrix} \int \frac{dx}{y} \\ \int \frac{x\,dx}{y} \end{pmatrix}$$

Ibiporanga: The moduli space of enhanced elliptic curves

$$y^2 = 4(x-t_1)^3 - t_2(x-t_1) - t_3$$

A matrix de conexão de Gauss-Manin



Moduli of (E, α_1, α_2) $\qquad$ E curva elíptica$/\mathbb{C}$

$$\begin{matrix}(E, & \alpha_1, & \alpha_2) \\ \wr\!\mid & \wr & \downarrow \\ (\tilde{E}, & \tilde{\alpha}_1, & \tilde{\alpha}_2)\end{matrix}$$

$\alpha_1, \alpha_2 \in H^1_{dR}(E) \cong 1$

Grothendieck 1966

$$H^1_{dR}(E) \times H^1_{dR}(E) \longrightarrow H^2_{dR}(E) \xrightarrow{\int_E}$$

$$\langle \alpha_1, \alpha_2 \rangle = \int_E \alpha_1 \wedge \alpha_2 = 1$$

α_1 é representado ~~por~~ 1-forma hol. em E.

~~A Samuel~~ Prop: Moduli $\cong \underset{(t_1,t_2,t_3)}{\mathbb{C}^3} \setminus \{\Delta = 0\}$

E_t ||

Algebra $(E, \alpha_1, \alpha_2) \longrightarrow \left(y^2 = 4(x-t_1)^3 - t_2(x-t_1) - t_3,\ \left[\frac{dx}{y}\right], \left[\frac{x\,dx}{y}\right]\right)$

~~Teoria~~ Teoria de formas quasi-modulares é $\cong$

$$\mathbb{C}\left[t_1, t_2, t_3, \frac{1}{\Delta}\right]. \qquad \underset{\mathrm{Im}(\tau)>0}{\mathbb{H}} \xrightarrow{\text{hol}} \mathbb{C} \qquad \in H^1_{dR}(E_t).$$

Atauúba: Ramanujan's vector field / Modular vector field.

$$\nabla \begin{bmatrix} \left[\frac{dx}{y}\right] \\ \left[\frac{x\,dx}{y}\right] \end{bmatrix} = A \begin{bmatrix} \left[\frac{dx}{y}\right] \\ \left[\frac{x\,dx}{y}\right] \end{bmatrix} \qquad T$$

$$T = \mathbb{C}^3$$

expli. A_{ij} - 1-formas em $(t_1, t_2, t_3) \in \mathbb{C}^3$

campos vectorias

Seja R um campo vectoral em

$$\exists!\, R \text{ em } T \qquad A(R) = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$$

$$R = R_1 \frac{\partial}{\partial t_1} + R_2 \frac{\partial}{\partial t_2} + R_3 \frac{\partial}{\partial t_3}$$

$$R = \left(t_1^2 - \frac{1}{12} t_2\right)\frac{\partial}{\partial t_1} + \left(4t_1 t_2 - 6t_3\right)\frac{\partial}{\partial t_2} + \left(6t_1 t_3 - \frac{1}{3}t_2^3\right)\frac{\partial}{\partial t_3}$$

$$\begin{cases} \dot{t}_1 = t_1^2 - \frac{1}{12} t_2 \\ \dot{t}_2 = 4t_1 t_2 - 6t_3 \end{cases}$$

Ramanuja:

$$E_{2k}(\tau) = 1 + (-1)^k \frac{4k}{B_k} \sum_{n \geq 1} \sigma_{2k-1}(n) q^n$$

Bernoulli

$$t_4 = \frac{\partial}{\partial \tau} t_1$$

$$\sigma_a(n) = \sum_{d|n} d^a$$

$$(t_1, t_2, t_3) = (a_1 E_2, a_2 E_4, a_3 E_6)$$ satisfaz R.

$$a_i = \left(\frac{2\pi i}{12}, 12\left(\frac{2\pi i}{12}\right)^2, \ldots\right).$$

$X \subseteq \mathbb{P}^N$

CY3

q_expanos.

$q = e^{2\pi i \tau}$

$\xrightarrow{hc_1} \mathbb{C}$