2 June 2020. (Letter to Jorge Duque)

Let us be given two linear dif. eq.

$$L_i : y_i^{(n_i)} = \sum_{j=0}^{n_i-1} a_{i,j}\, y^{(j)} \qquad i=1,2$$

compute the linear diff. equ. of $f_1 y_1 + f_2 y_2$ where f_1, f_2 are two rational functions in z

1. Write in the syst. format

$$Y_i' = \begin{pmatrix} y_i \\ y_i' \\ \vdots \\ y_i^{(n_i-1)} \end{pmatrix}' = \underbrace{\left(\begin{array}{c|c} 0 & I \\ \hline a_{i,0} & a_{i,1}, \cdots\ a_{i,n_i-1} \end{array}\right)}_{A_i} Y_i$$

$$Z_i = \underbrace{\begin{pmatrix} f_i, & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ & & 1 & \\ & 0 & & 1 \end{pmatrix}}_{B_i} Y_i \qquad Z_i' = \underbrace{(B_i' B_i^{-1} + B_i \cdot A_i\, B_i^{-1})}_{\text{Compute this}} Z_i$$

2. If $n_1 < n_2$ then Substitute Z_i with $\begin{pmatrix} Z_i \\ 0 \\ 0 \end{pmatrix}_{n_2 \times 1}$

A_i " $\begin{pmatrix} A_i & 0 \\ 0 & 0 \end{pmatrix}_{n_2 \times n_2}$

Now $Z_i' = A_i Z_i$ $i=1,2$ have dim $= n_2 = n_1$

3.

$$W = \begin{bmatrix} Z_1 + Z_2 \\ Z_1 \\ Z_2 \end{bmatrix} \qquad W' = \begin{pmatrix} 0 & A_1 & A_2 \\ 0 & A_1 & 0 \\ 0 & 0 & A_2 \end{pmatrix} W$$

compute the linear diff equation of the first entry of W. (Note that this is $f_1 y_1 + f_2 y_2$).

For this I have written the code sysdif in foliation.lib