Jorge Thesis. 14/03/2020

f tame polynomial $L=\{f=0\}$, X compactification and desingular.

$$H^n_{dR}(X) \to H^n_{dR}(U)$$

$$H_n(X) \leftarrow H_n(U)$$

$$H^n_{dR}(X)_0 = \mathrm{Im}\left(H^n_{dR}(X) \to H^n_{dR}(U)\right) \subseteq H^n_{dR}(U)$$

$$H_n(X)_0 = H_n(U) / \ker\left(H_n(X) \leftarrow H_n(U)\right)$$

we have a well-defined map

$$\begin{aligned} H^n_{dR}(X)_0 \times H_n(X)_0 &\to \mathbb{C} \\ (\omega, \delta) &\mapsto \int_\delta \omega \end{aligned}$$

which is perfect (verify this). Therefore $H_n(X)_0$ is dual to $H^n_{dR}(X)_0$

In order to compare it with Steenbrink we consider

$$\begin{tikzcd} X \arrow[r, "desing"] & D = \tilde{D}/G & \tilde{D} \arrow[l, "q.m"'] \\ U \arrow[u, phantom, "\subseteq"] \arrow[r, equal] & U \arrow[u, phantom, "\subseteq"] & \tilde{U} \arrow[u, phantom, "\subseteq"] \arrow[l] \end{tikzcd}$$

Note that $\tilde{D} - \tilde{U}$ is the usuall transversal hyperplane section.

$$\begin{tikzcd} H^n_{dR}(X) \arrow[d, "\alpha"] & H^n_{dR}(D) := H^n_{dR}(\tilde{D})_G \arrow[l] \arrow[r, hook] \arrow[d, "\tilde{\beta}"] & H^n_{dR}(D) \arrow[d, "\beta"] \\ H^n_{dR}(U) \arrow[r, equal] & H^n_{dR}(U) := H^n_{dR}(\tilde{U})_G \arrow[r, hook] & H^n_{dR}(\tilde{U}) \end{tikzcd}$$

this is definition

one has to verify this. It is not def.

Prop: $\mathrm{Im}(\alpha) = \mathrm{Im}(\hat{\beta})$

Proof: $\mathrm{Im}(\tilde{\beta}) \subseteq \mathrm{Im}(\alpha)$ is trivial.

For $\mathrm{Im}(\alpha) \subseteq \mathrm{Im}(\tilde{\beta})$

Both $\mathrm{Im}(\alpha)$ and $\mathrm{Im}(\beta)$ are charcterized by the fact that the integration of their elements over cycles at infinity is zero.

Cycle at infinity = cycles whose support can be out of any compact subset of the ambient space (U and $\tilde{U}$).

If $\omega \in \mathrm{Im}(\alpha)$ then $\int_{\text{cycles at infinity}} \omega = 0$. ω can be considered as an element in $H^n_{dR}(\tilde{U})$ and its integration over cycles at ∞ is zero $\Rightarrow$ $\alpha \in \mathrm{Im}(\beta)$.