

Title: Cox rings of K3 surfaces.

Abstract. Let X be a smooth projective variety over the complex numbers with finitely generated divisor class group $\text{Cl}(X)$. The Cox ring of X is the $\text{Cl}(X)$ -graded algebra defined as [4]:

$$R(X) = \bigoplus_{[D] \in \text{Cl}(X)} \Gamma(X, \mathcal{O}_X(D)).$$

When $R(X)$ is finitely generated, the variety X can be obtained as a quotient of an open subset of $\text{Spec } R(X)$ by the action of a quasi-torus and this provides X with a set of global homogeneous coordinates. The two main problems in this setting are: classify varieties with finitely generated Cox ring (called Mori dream spaces after [6]) and compute the Cox ring of such varieties.

In this talk we will deal with both problems in case X is a projective K3 surface. In [3] we proved that Mori dream K3 surfaces are exactly those having finite automorphism group and we computed $R(X)$ for some classes of examples. In [1] we developed a different technique which allowed to prove that the degrees of a generating set of the Cox ring of a K3 surface are either classes of smooth rational curves, sums of at most three elements of the Hilbert basis of the nef cone, or of the form $2(E + E')$, where E, E' are the classes of two genus one curves with $E \cdot E' = 2$ (see [5, 1]). In case $R(X)$ is finitely generated, this provides a finite set of degrees for the generators of the ring. We will describe these results and their application to the study of Cox rings of Mori dream K3 surfaces with Picard number three and four [1, 2].

This is joint work with Claudia Correa Deisler (Universidad de Tarapacá, Chile), Antonio Laface (Universidad de Concepción, Chile) and Xavier Roulleau (Aix-Marseille Université, France).

References

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