12/10/2021

Dear Movasati,

So far, I cannot make sense of (1) in Th1: to make sense of $\frac{\omega}{\partial s}$, you must use a specific description of cohomology – something I hate. It is also unclear to me where rational curve is used (as opposed to fixed genus).

About cycles: it is true that flat family is too much to ask, but for any family, parametrized by S, it will hold over some Zariski open. In your situation, it hence suffices.

I wonder whether what you do is related to the following picture. Suppose Z_s is an algebraic ~~base~~ family of cycles on X, codimension m, connected parameter space S. Then $Z_t - Z_s$ is cohomologous to 0, and has a class in the intermediate jacobian $J = H^{2m-1}(X,\mathbb{Z})\backslash H^{2m-1}(X,\mathbb{C})/F^m$.
We also have the jacobian variety J_S of S [compactified, $\bar{S}$ non singular] : same for $\bar{S}$, ~~with~~ for points in $\bar{S}$]
and we have $J_S \longrightarrow J$, class$(Z_t - Z_s)$ image of class of $t - s$. In the Hodge structure language, we will have inside $H^{2m-1}(X,\mathbb{C})$ a sub Hodge structure $\mathcal{H}$ of type $\{(m-1,m)(m,m-1)\}$, and

Related to prop 2 p 6 ?: but your speculation not involving $\mathcal{H}$ strange — if Grothendieck: the Hodge conjecture is false ...

the class of $Z_t - Z_s$ will be in the image

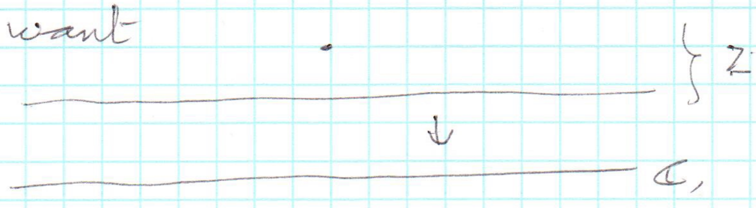
$$\mathcal{H}_Z \backslash \mathcal{H}_C / F^m \hookrightarrow J$$

(an abelian variety contained in the intermediate jacobian).

For odd dimentional hypersurfaces X in $\mathbb{P}^N$, dim $2m-1$ and except when H^{2m-1} is of type $\{(m-1, m)(m, m-1)\}$, for general X, the largest sub ~~Hog~~ Hodge structure of H^{2m-1} of type $\{(m-1, m)(m, m-1)\}$ should be reduced to 0. This is similar to the fact that for general X of dimension $2m$, the only Hodge cycles in $H^{2m}(X)$ are the obvious ones — except when H^{2m} is purely of type $\{(m, m)\}$.

In our case, this implies that for general X, algebraic equivalence implies same class in the intermediate jacobian.

On p1, your notion of algebraic family is unreasonable. Flat might be too much, but you don't want



which is proper. Your def 1 p 4 is bizarre.

Best

P.