① Find the Jordan form and Jordan basis for

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

② Let $A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{pmatrix}$.

Is A similar to $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$?

③ Let V be a finite dimensional vector space over $\mathbb{C}$.

Let $T: V \to V$ be a linear operator. Suppose $\lambda = 0$ is the only eigenvalue of T. Show that T is nilpotent (ie, $\exists k$ st. $T^k = 0$).

(Hint: Use Cayley-Hamilton).

④ Let $A \in M_{n \cdot n}(\mathbb{C})$. Suppose $\lambda_1 \ldots \lambda_k$ are the distinct eigenvalues of A, with multiplicities $m_1, \ldots, m_k$.

Prove that

$$\det(A) = \lambda_1^{m_1} \ldots \lambda_k^{m_k}$$