MATH 110 — Worksheet #2

(1) Let V be an inner product space (denote the inner product by $\langle \cdot,\cdot \rangle$).

(a) Let $S \subseteq V$ be a subset. Prove that $S^{\perp}$ is a subspace of V.

(b) Prove that $S^{\perp} = (\mathrm{span}(S))^{\perp}$

(c) Let $S_0 \subseteq V$ be another subset. Show that if $S_0 \subseteq S$, then

$S^{\perp} \subseteq S_0^{\perp}$.

(d) Let $W \subseteq V$ be a finite dimensional subspace of V.

Prove that $W = (W^{\perp})^{\perp}$.

(2) Let V be an inner product space and $T, U : V \longrightarrow V$ be linear operators on V. Show the following:

(a) $(T+U)^* = T^* + U^*$

(b) $(cT)^* = \bar{c}\, T^*$, $c \in F$

(c) $(TU)^* = U^* T^*$

(d) $(T^*)^* = T$

(e) $(Id)^* = Id$