MATH 110: WORKSHEET

① Say whether the expression defines an inner product:

ⓐ $V = \mathbb{R}^n$, $\quad x = (x_1 \dots x_n)$, $y = (y_1, \dots y_n)$

$$\langle x, y \rangle = \sum_{i=1}^{n} c_i x_i y_i \quad , \quad c_i > 0 \quad i = 1 \dots n$$

ⓑ $V = \mathbb{R}^2$, $\quad x = (x_1, x_2)$ $\quad y = (y_1, y_2)$

$$\langle x, y \rangle = x_1 x_2 - y_1 y_2$$

ⓒ $V = C([0,1]) = \{ f : [0,1] \to \mathbb{R} \, / \, f \text{ Continuous} \}$

$$\langle f, g \rangle = \int_0^1 f(t) g(t) \, dt$$

ⓓ $V = C([0,1])$

$$\langle f, g \rangle = \int_0^{1/2} f(t) g(t) \, dt$$

ⓔ $V = C([0,1])$,

$$\langle f, g \rangle = \int_0^1 f'(t) g(t) \, dt$$

(2) Let V be an inner product space.

Let $x, y \in V$ and suppose x and y are orthogonal.

Prove that $\|x+y\|^2 = \|x\|^2 + \|y\|^2$.

What does this mean geometrically in $\mathbb{R}^2$?

(3) Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$,

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3,\ x_1 + 3x_3;\ x_2 - x_3)$$

Find an orthonormal basis for $R(T)$.

(4) Let $x, y \in V$ (V vector space over F), $\langle \cdot,\cdot \rangle$ inner product on V

Prove:

(a) If $x = cy$, $c \in F$, then $|\langle x, y\rangle| = \|x\| \|y\|$.

(b) If $|\langle x, y\rangle| = \|x\| \|y\|$, then $\exists\, c \in F$ s.t. $x = cy$.