HOMEWORK 5

1. **Exercise.** Let V be a conformal vertex algebra. Show that

$$[L_n, Y(a,w)] = \sum_{m\geq -1} \binom{n+1}{m+1} Y(L_m a, w) w^{n-m}$$

2. **Exercise.** Let $v(t)\partial_t = v_1 t^2 \partial_t + v_2 t^3 \partial_t + \dots$ be a derivation of $\mathscr{O} = k[[t]]$. Consider the automorphism of $\mathscr{O}$:

$$t \mapsto \rho(t) = \exp\left(v(t)\partial_t\right) v_0 \cdot t = v_0 t + v_1 v_0 t^2 + \dots$$

for $v_0 \in k^*$. Let $R(\rho)$ be the automorphism of V given by

$$R(\rho) = \exp\left(-\sum_{i>0} v_i L_i\right) v_0^{-L_0}$$

Show that $R(\tau \circ \rho) = R(\rho)R(\tau)$ where $\tau \circ \rho$ is the composition of the automorphisms.

3. **Exercise.** Let V be a vertex algebra and $a, b, c \in V$. Consider a linear functional $\varphi \in V^*$. Show that

$$\varphi\Big(Y(a,z)Y(b,w)c\Big) \in k((z))((w))$$
$$(-1)^{ab}\varphi\Big(Y(b,w)Y(a,z)c\Big) \in k((w))((z))$$
$$\varphi\Big(Y\big(Y(a,z-w)b,w\big)c\Big) \in k((w))((z-w))$$

are three expansions of the same element in $V[[z,w]][z^{-1}, w^{-1}, (z-w)^{-1}]$.

4. **Exercise.** Consider the $bc - \beta\gamma$ system. This is a vertex algebra generated by 4 fields. bc are odd, $\beta\gamma$ are even and their brackets are given by

$$[\beta_\lambda \gamma] = 1, \qquad [b, c] = 1$$

Find a Virasoro vector L such that γ, c are primary of conformal weight 0 and β, γ are primary of conformal weight 1 with respect to L.

Date: Due: 15/4/2014.