

# Homework 5

Due 23 September 2019

**Exercise 1.** Let  $\mathcal{L}$  be the tautological line bundle over  $M = \mathbb{R}P^n$  whose fiber over a point  $p$  in  $\mathbb{R}P^n$  parametrizing the line  $\ell \in \mathbb{R}^{n+1}$  is given by  $\mathcal{L}_p = \ell$ .

1. Compute the transition functions for  $\mathcal{L}$  for any cover of  $\mathbb{R}P^n$ .
2. Show that  $\mathcal{L}$  is a subbundle of a trivial vector bundle over  $\mathbb{R}P^n$ .
3. Is it true that for any line bundle  $\mathcal{L}$  over a manifold  $M$ , there exists a trivial vector bundle  $E$  over  $M$  such that  $\mathcal{L} \subset E$  is a subbundle?

**Exercise 2.** Compute the transition functions for  $TS^2$  associated with the two local trivializations determined by the stereographic projections.

**Exercise 3.** Let  $E$  be a rank 2 vector bundle over  $S^3$ . And let  $E^*$  be its dual vector bundle. Show that  $F = E \otimes E^*$  is a rank 4 vector bundle over  $S^3$  and that there exists a trivial line subbundle of  $F$ .

**Exercise 4.** Show that  $S^3$  is parallelizable.