Homework 2

Due 16 September 2019

Exercise 1. *Let $F : \mathbb{R} \to \mathbb{R}^2$ be defined by $F(t) = (\cos t, \sin t)$. Let $X = \frac{\partial}{\partial t} \in \mathfrak{X}(\mathbb{R})$ and*

$$Y = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \in \mathfrak{X}(\mathbb{R}^2).$$

Show that for each $p \in \mathbb{R}$ we have $F_* X_p = Y_{F(p)}$.

Exercise 2. *Let $F : M \to N$ be a smooth map with M connected and $dF = 0$. Show that F is constant.*

Exercise 3. *Let $M_1, \cdots, M_k$ be smooth manfifolds. Let $\pi_i : M_1 \times \cdots \times M_k \to M_i$ be the projection to the i-th factor. Show that for a collection of points $p_i \in M_i$, $i = 1, \cdots, k$ the map*

$$\alpha : T_{(p_1,\dots,p_k)} \left(M_1 \times \cdots \times M_k\right) \to T_{p_1} M_1 \oplus \cdots \oplus T_{p_k} M_k,$$

defined by

$$\alpha(X) = \left(\pi_{1*} X, \cdots, \pi_{k*} X\right)$$

is an isomorphism.

Exercise 4. *Let G be a Lie group*

1. *Let $m : G \times G \to G$ be the multiplication map. Identifying $T_{(e,e)}(G \times G) \simeq T_e G \oplus T_e G$ as above. Show that $m_* : T_e G \oplus T_e G \to T_e G$ is given by $m_*(X, Y) \simeq X + Y$.*
2. *Let $i : G \to G$ be the inversion map. Show that $i_* = -\mathrm{Id}_{T_e G}$.*

Exercise 5. *Let $F : M \to N$ be a smooth map. Show that $dF : TM \to TN$ is smooth.*