Homework 1

Due August 19 2019

Exercise 1. *Prove that* $\mathbb{R}P^2$ *is Hausdorff.*

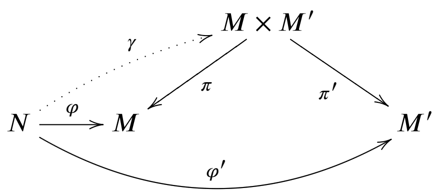
Exercise 2. *Prove that* $\mathbb{R}P^2$ *is compact*

Exercise 3. *Let* M *be a second countable Hausdorff topological space. Show that the following are equivalent*

1. *For every point* $m \in M$ *there exists an open subset* $m \in U \subset M$ *and an open subset* $V \subset \mathbb{R}^n$ *together with an homeomorphism* $\varphi : U \xrightarrow{\sim} V$.

2. *For every point* $m \in M$ *there exists an open subset* $m \in U \subset M$ *and an homeomorphism* $\varphi : U \xrightarrow{\sim} \mathbb{R}^n$.

Exercise 4. *Let* M, M' *and* N *be topological manifolds. Let* $\varphi : N \to M$ *and* $\varphi' : N \to M'$ *be continuous maps. Show that there exists a unique continuous map* $\gamma : N \to M \times M'$ *making the following diagram commutative:*



Exercise 5. *Does there exists a non-constant and non-surjective continuous map* $\varphi : S^1 \to S^1$*?*

Exercise 6. *Let* $\mathbb{T}^2$ *be the two-torus. Find a continuous map* $\varphi : \mathbb{R}^1 \to \mathbb{T}^2$ *such that* $\varphi(\mathbb{R}^1) \subset \mathbb{T}^2$ *is dense. Does there exists a similar map from* S^1 *instead of* $\mathbb{R}^1$*?*

Exercise 7. *Let* $f : M \to N$ *a continuous map and let* $n \in N$ *be a point in its image. Is* $f^{-1}(n) \subset M$ *a topological manifold?*