Test 1

Most exercises from Artin

Sept 27
Test time: 5hs

All fields of characteristic zero.

1 Exercise. Let α be a complex root of the polynomial $x^3 + x + 1$ over $\mathbb{Q}$ and let K be the splitting field of this polynomial over $\mathbb{Q}$.

- Is $\sqrt{-3}$ in the field $\mathbb{Q}(\alpha)$? is it in K?
- Prove that the field $\mathbb{Q}(\alpha)$ has no automorphism except for the identity.

2 Exercise. Let K/F be a Galois extension whose Galois group is the symmetric group S_3. Is it true that K is the splitting field of an irreducible cubic polynomial over F?

3 Exercise. Let $F \subset K$ be a field extension of degree 5. Let $\alpha \in K$ generate the extension. Prove that α^2 generates the same extension.

4 Exercise.

a) Let $f(x) = x^4 + 3x^3 + 7x^2 + 8x + 6$. And let $F = \mathbb{Q} \subset K$ be a splitting field of f. Find $Gal(K/F)$.

b) Find the Galois group of $\mathbb{Q}\left(\sqrt{2+\sqrt{2}}\right)/\mathbb{Q}$.

c) Is every degree 4 extension of $\mathbb{Q}$ Galois?