Homework 4

All exercises from Artin

due Sept 27

1 Exercise. Let α be a complex root of the polynomial $x^3 + x + 1$ over $\mathbb{Q}$ and let K be the splitting field of this polynomial over $\mathbb{Q}$.

- Is $\sqrt{-3}$ in the field $\mathbb{Q}(\alpha)$? is it in K?
- Prove that the field $\mathbb{Q}(\alpha)$ has no automorphism except for the identity.

2 Exercise. Let $f(u)$ be a symmetric polynomial of degree d in $u_1, \cdots, u_n$ and let $f^0(u_1, \cdots, u_{n-1}) = f(u_1, \cdots, u_{n-1}, 0)$. Say that $f^0(u) = g(s^0)$ where s_i^0 are the elementary symmetric functions in $u_1, \cdots, u_{n-1}$. Prove that if $n > d$ then $f(u) = g(s)$.

3 Exercise. Prove that the square of the determinant of the Vandermonde matrix is the discriminant.

4 Exercise. Let $\alpha = \sqrt[3]{2}$, $\zeta = \frac{1}{2}(-1 + \sqrt{-3})$, $\beta = \alpha\zeta$. PRove that for all $c \in \mathbb{Q}$, $\gamma = \alpha + c\beta$ is the root of a degree six polynomial of the form $x^6 + ax^3 + b$

5 Exercise. Let K/F be a Galois extension whose Galois group is the symmetric group S_3. Is it true that K is the splitting field of an irreducible cubic polynomial over F?