Homework 3

All exercises from Artin

due Sept 2

1 Exercise. Show that the difference of two roots α, β of the polynomial $x^q - x$ is a root of the same polynomial.

2 Exercise. The polynomials $f = x^3 + x + 1$ and $g = x^3 + x^2 + 1$ are irreducible over $\mathbb{F}_2$. Let K be the field extension obtained by adjoining a root of f and let L be the extension obtained by adjoining a root of g. Describe explicitly the isomorphism $K \simeq L$.

3 Exercise. Determine the irreducible polynomial for $i + \sqrt{2}$ over $\mathbb{Q}$.

4 Exercise. Determine the intermediate fields between $\mathbb{Q}$ and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$.

5 Exercise. Let $K = \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ Determine $[K : \mathbb{Q}]$ and prove that K is a Galois extension of $\mathbb{Q}$ and determine its Galois group.