

# Homework 2

All exercises from Artin

due Aug 24

**1 Exercise.** Express  $\cos 15^\circ$  in terms of square roots, even better if you trisect the  $45^\circ$  angle and construct this cosine therefore.

**2 Exercise.** Assuming that  $\pi$  is transcendental. Show that it's impossible to construct a square whose area is the same as the area of the unit circle.

**3 Exercise.** Prove that it is impossible to construct the side length of a cube whose volume is 2.

**4 Exercise.** Characterize the constructible real numbers in the case that three points are given in the plane to start.

**5 Exercise.** For which fields  $F$  and which primes  $p$  does  $x^p - x$  have multiple roots?

**6 Exercise.** Let  $F$  be a field of characteristic  $p$ . Factor the polynomial  $x^p + 1$  in irreducible factors in  $F[x]$ .

**7 Exercise.** Let  $\alpha_1, \dots, \alpha_n$  be roots of a polynomial  $f \in F[x]$  of degree  $n$  in an extension field  $K$ . Find the best upper bound that you can for  $[F(\alpha_1, \dots, \alpha_n) : F]$ .