

Homework 1

All exercises from Artin

due Aug 17

- 1 **Exercise.** Let F be a field Find all elements $a \in F$ such that $a = a^{-1}$.
- 2 **Exercise.** Let F be a field contained in an integral domain R . Suppose that $\dim_F R < \infty$. Prove that R is a field.
- 3 **Exercise.** Let F be a field with exactly 8 elements. Is the characteristic of F 8?
- 4 **Exercise.** Let $K = F(\alpha)$ where α is a root of the irreducible polynomial $f(x) = x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$. Determine α^{-1} explicitly in terms of α and the coefficients a_i .
- 5 **Exercise.** Let F be a field and α an element which generates a field extension of F of degree 5. Prove that α^2 generates the same extension.
- 6 **Exercise.** Let $\zeta = e^{2\pi i/7}$, $\eta = e^{2\pi i/5}$ show that $\eta \notin \mathbb{Q}(\zeta)$.
- 7 **Exercise.** Let $\alpha, \beta \in \mathbb{C}$. Prove that if $\alpha + \beta$ and $\alpha\beta$ are algebraic numbers, then α and β are algebraic.