Lista 1

Entregar 28/3/2023

1 Exercise. Let C be a category with a zero object and direct sums as defined in class. For $f, g \in \mathrm{Hom}_C(X, Y)$ we defined $f + g \in \mathrm{Hom}_C(X, Y)$ as the composition

$$X \xrightarrow{\Delta} X \oplus X \xrightarrow{f,g} Y \oplus Y \xrightarrow{\nabla} Y.$$

Show that it is associative, commutative and that 0 is the unit for addition.

2 Exercise. Let $C_n = \mathbb{Z}/8\mathbb{Z}$ for $n \geq 0$ and $C_n = 0$ for $n < 0$. Let $d_n(x) := 4x$. Show that $C_\bullet$ is a complex and compute its homology.

3 Exercise. Let $\mathcal{A}$ be an abelian category. Show that $H_n : Ch(\mathcal{A}) \to Ab$ is a functor from chain complexes to abelian groups.

4 Exercise. Write down the complex computing the singular homology of the tetrahedron as computed in class (notice that I included a 3-simplex, unlike Weibel). Compute its homology.

5 Exercise. Find an example of an abelian category $\mathcal{A}$ and an object $X \in \mathcal{A}$ which is not projective.

Find an example of an abelian category without enough projectives.