Calculus D, Homework 2
Due Friday July 16th

Exercise 1. Find the derivatives of the following functions:

$$f(x) = \sin^{-1}(e^x), \tag{1a}$$
$$g(x) = \log(\sin(x)e^x), \tag{1b}$$
$$h(x) = \frac{e^{\sqrt{x}}}{\sqrt{1 + \log(x)^2}}, \tag{1c}$$
$$i(x) = e^{-|x|}. \tag{1d}$$

Can you tell where are these functions defined?

Exercise 2. A woman is at a point A on the shore of a circular lake with radius 2mi and wants to be at the point C diametrically oposed to A, in the shortest time possible. She can walk at the rate of $4mi/h$ and row a boat at a rate of $v\ mi/h$. At what angle θ to the diameter $\overline{AC}$ should she row?. The answer should be in therms of v. Are there values of v such that it's better for the woman to just row or just walk?.

Exercise 3. Show that among all triangles with two perpendicular sides and given perimeter, the ones with greatest area are isosceles. Compare with exercise 3 for group C, this should be much harder.

Exercise 4. A Boat leaves a dock at 2:00 pm and travels due south at a speed of $20km/h$. Another boat has been heading due east at $15km/h$ and reaches the same dock at 3:00 pm. At what time were the two boats closest together?

Exercise 5. Find the local maximums and minimums of the function $\sin(1/x)$ defined for all $x \neq 0$.

Exercise 6. Find the maximum of the function e^{-x^2} defined for all real x. What about the minimum?.

Find the maximum of the function $e^{-|x|}$ defined for all real x. What about the minimum?

Exercise 7. Two runners start a race at the same time and finish in a tie. Show that at some point they had the same velocity.

Exercise 8. Find the point P in the hyperbola $xy = 8$ that is closest to the point $Q = (3, 0)$. Is the line $\overline{PQ}$ tangent to the hyperbola at P?, is it perpendicular to the tangent line at P?

Exercise 9. The degree of a polynomial $P(x)$ is the highest power of x appearing in P. Prove that the following polynomial of degree 7 has exactly one real root:

$$P(x) = x^7 + 5x^3 + x - 6. \tag{2}$$

Can you prove that every polynomial of odd degree has at least one real root? (note that if you want to do this seriously is not so obvious, you may use the fact that for odd n the limits of x^n as $x \to \pm\infty$ are $\pm\infty$ respectively, can you prove a similar statement for every polynomial of odd degree?).