

# GrabCut+D

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  - ▶ Second: Probabilistic Model.
  - ▶ Third: Robust Depth Based Seeding

# GraphCut

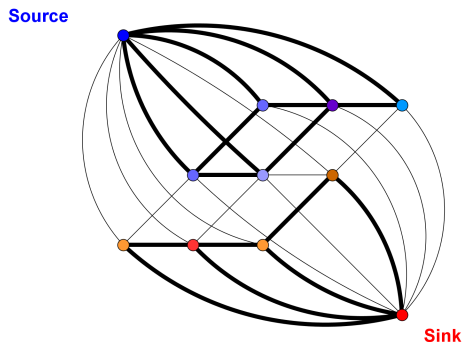
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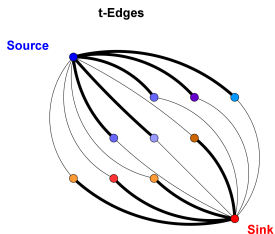
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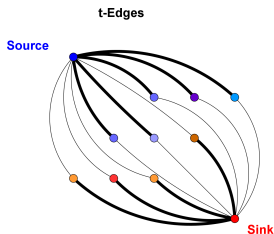
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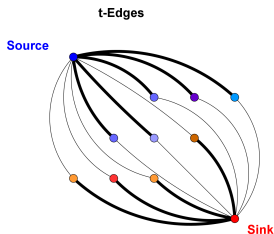


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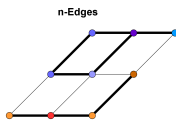
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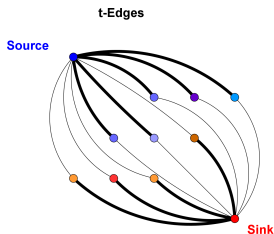
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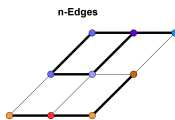
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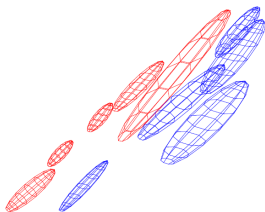
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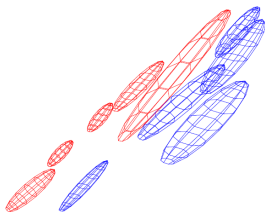
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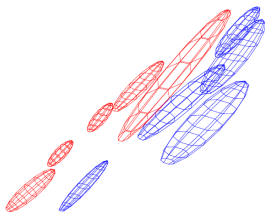
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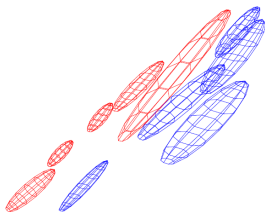
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Initial Selection



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Further Interaction



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Final Result



# First Approach: Region Growing



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**Main Idea:** Extract an object from an image implementing a priority search algorithm to identify its connected-depth component. Then, apply GrabCut in the contour of the component to improve the result.

# First Phase

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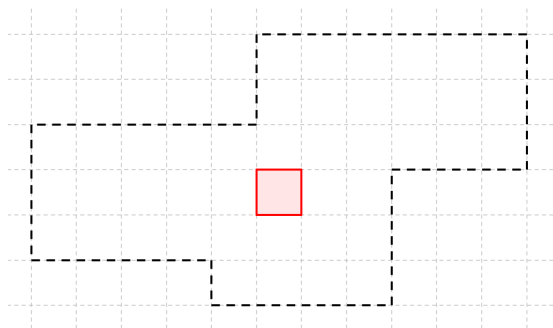
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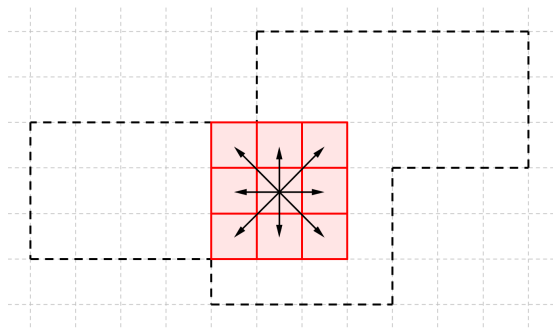
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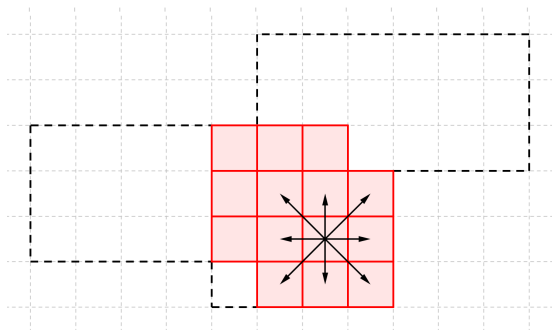
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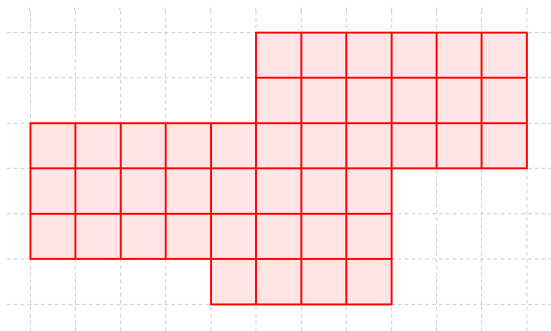
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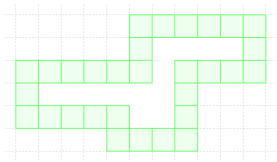
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- 1 Define a band around the *contour pixels* of the component.

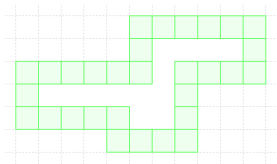
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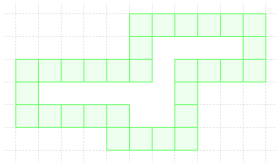
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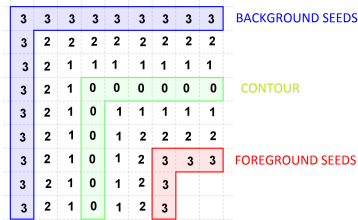
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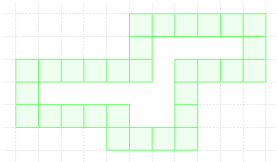


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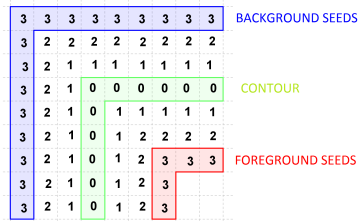


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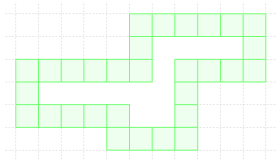


- 3 Apply Grabcut on the 8-connected graph associated to the band.

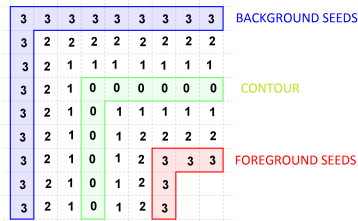


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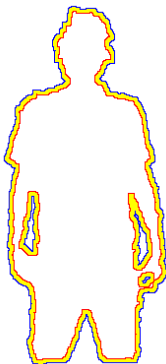
RGB



Depth



Graph



Result



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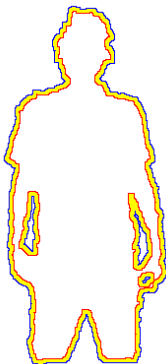
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Depth data must be improved!

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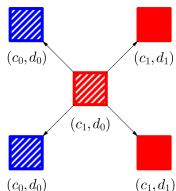
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- If there is depth disparity with at least two of them, then compare the colour values.

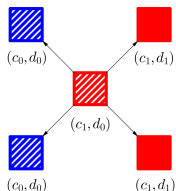
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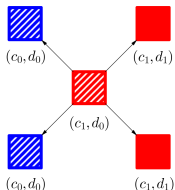
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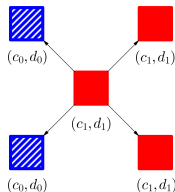
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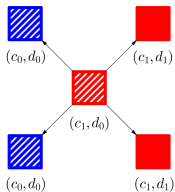


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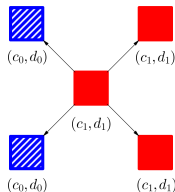


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# Result of Color Based Depth Filter

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Filter



Previous Result



New Result



Detail



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- Pros:
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  - 3 Accurate Colour Models
- Cons:
  - 1 Vulnerable to depth noise.
  - 2 No useful in the case of object-background adjacency.

## Second Approach: Probabilistic Model

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**Main Idea:** Extend the energy function model implemented in GrabCut to include the depth data in analogous way to colour. Construct Gaussian Mixture Models associated to FG and BG depth and use them to define the seeding.

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- Depth model  $\omega_D$  is constructed from Gaussian Mixtures.

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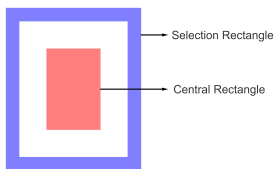
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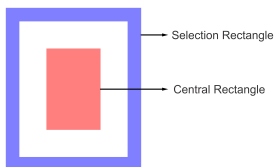
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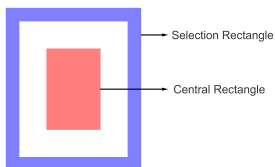
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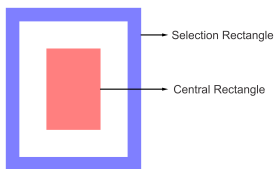


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- 3 Discard the Gaussian components of FGDM whose mean value have the highest probability according to BGDM.



# Seeding

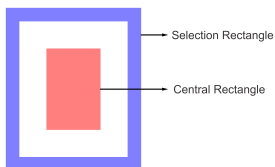
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- 3 Discard the Gaussian components of FGDM whose mean value have the highest probability according to BGDM.
- 4 Evaluate all pixels in the image respect to these models, and mark them as FG Seeds or BG Seeds according to the obtain likelihood.

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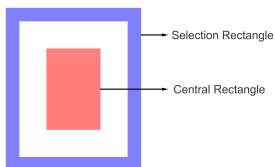
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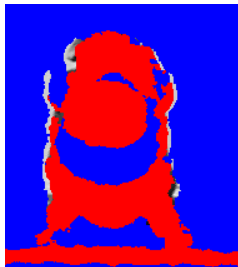


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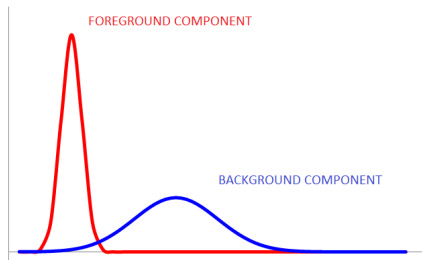
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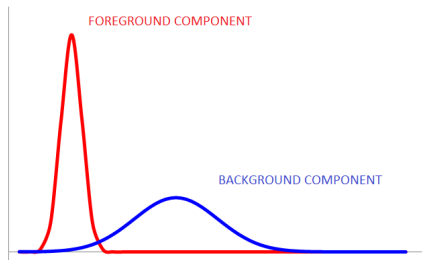


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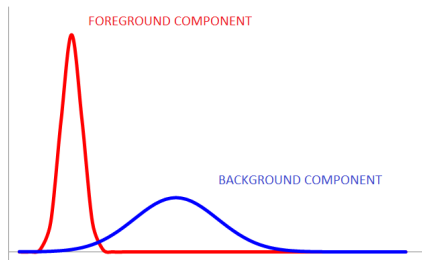


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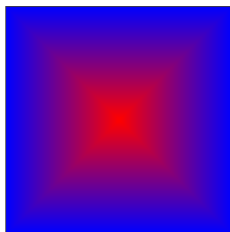
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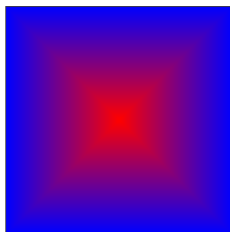
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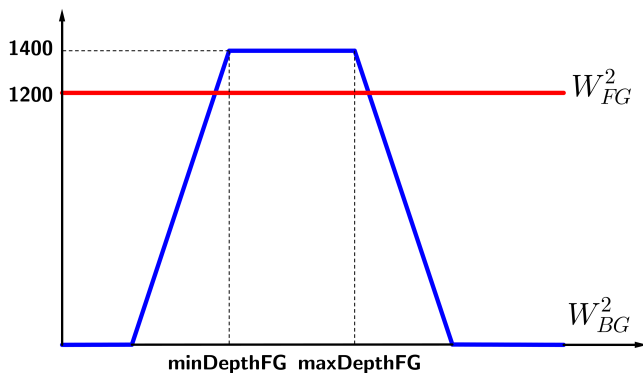
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- 8 For all  $p$  set  $W_{FG}^2(p) = 1200$ .
- 9 Set  $W_{BG}^2$  a stepwise linear function that separates between close pixels and far pixels respect to FG clusters.



# Results: Central and Closest is Foreground

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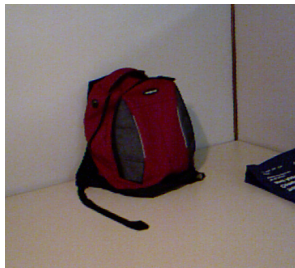
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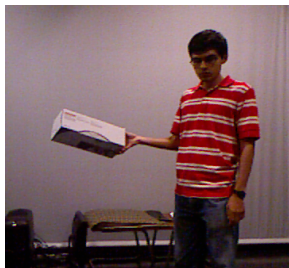
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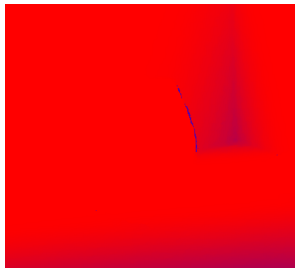




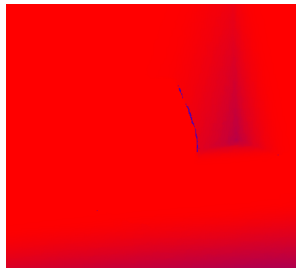
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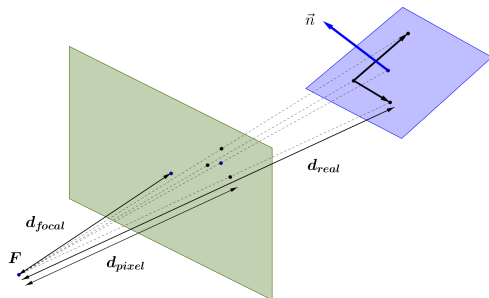


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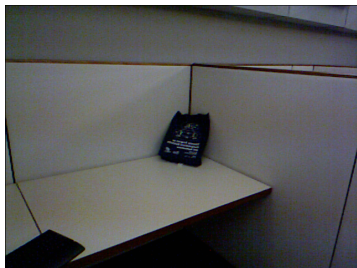
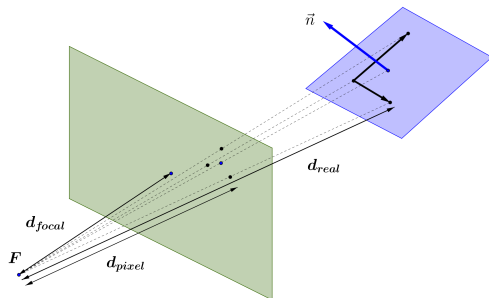
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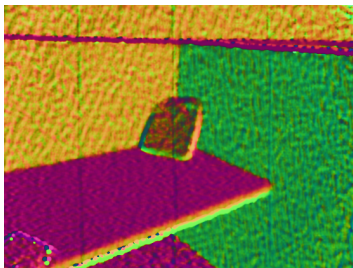
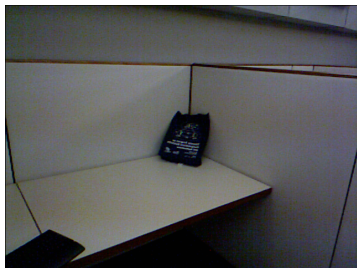
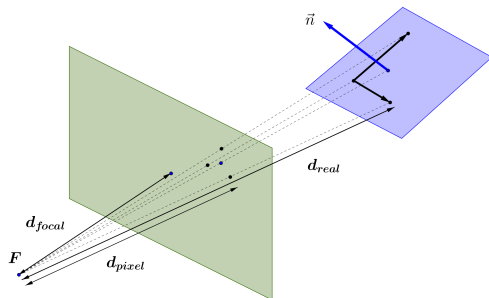
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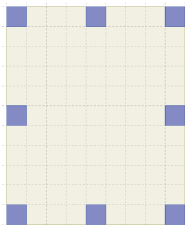


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- 2 Take a sample of pixels from the border of the Selection Rectangle and store them in a queue.

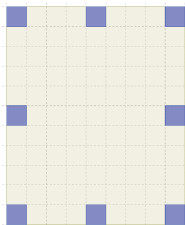
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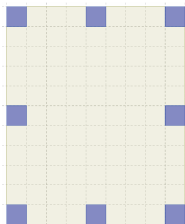
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- Fix parameters for normal tolerance  $e_n$  and color tolerance  $e_c$ .
- Pick the first pixel of the queue  $p$  and identify the connected set of pixels around  $p$  satisfying  $\|n_p - n_x\| < e_n$  and  $\|c_p - c_x\| < e_c$ . Call this set of pixels  $PN(p)$ , and label all these pixels as *belong Component*.

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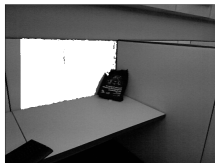
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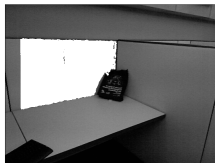
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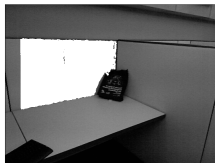
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- 7 Set  $W_{BG}^3(p) = 0$  for all pixels  $p$ . Set  $W_{FG}^3(p) = 1200 \iff p$  is *planar Pixel*.

# Results: External and Planar is Background

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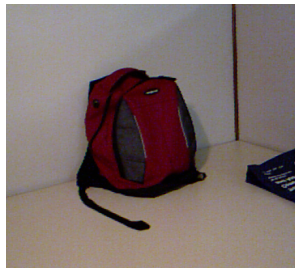
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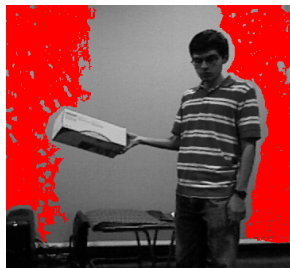
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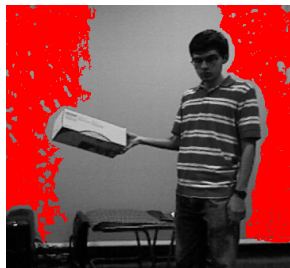
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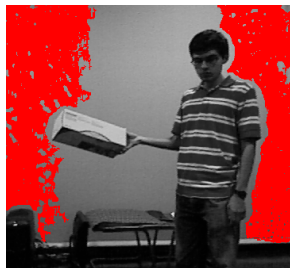


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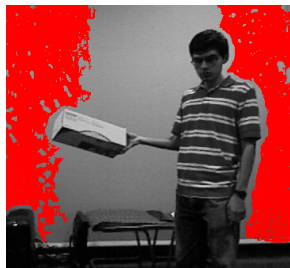
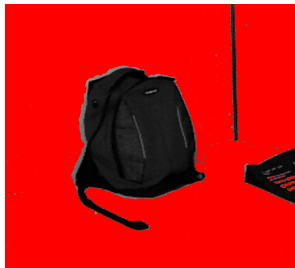




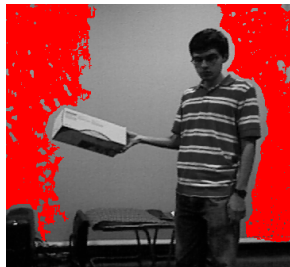
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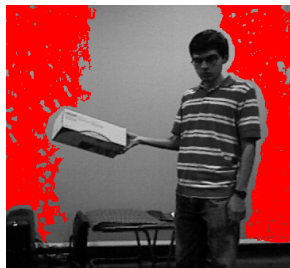
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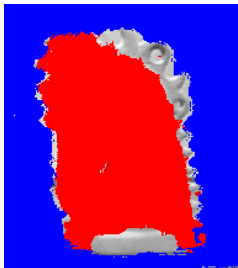
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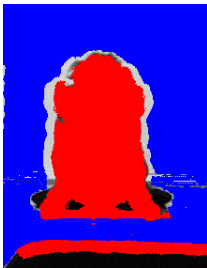
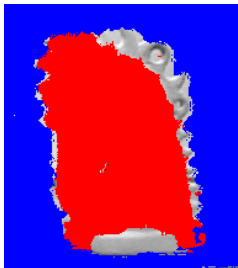
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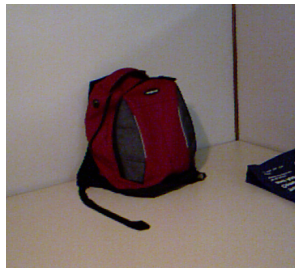
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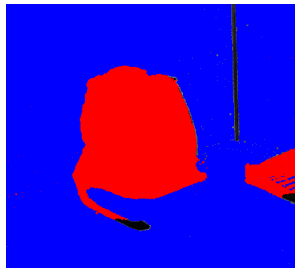
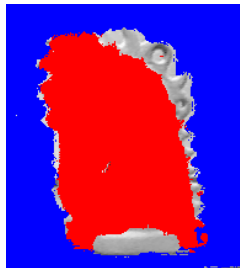
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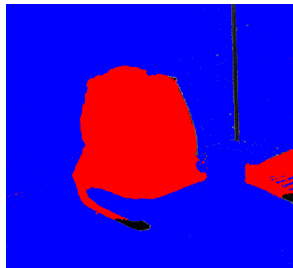
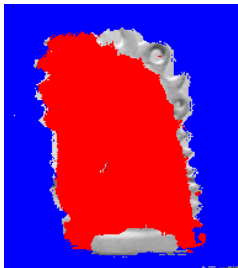


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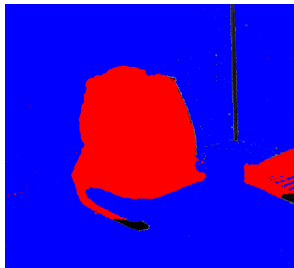
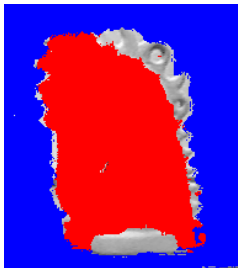




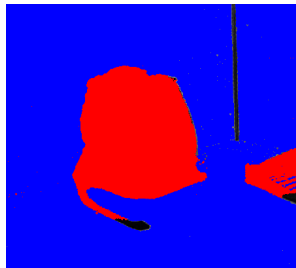
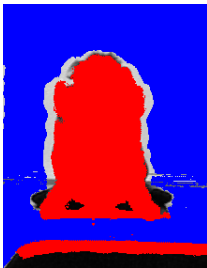
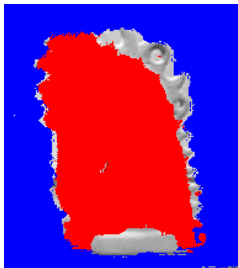
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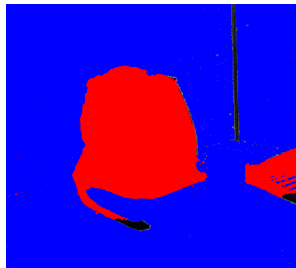
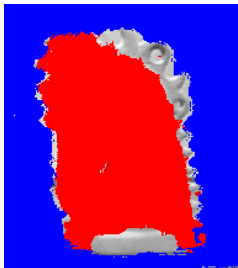
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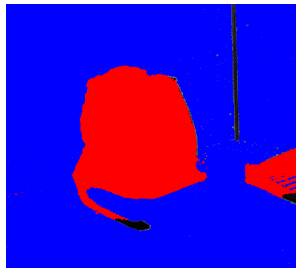
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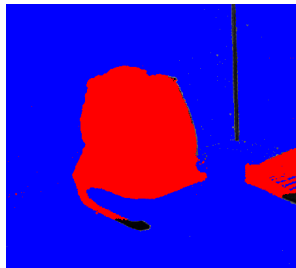
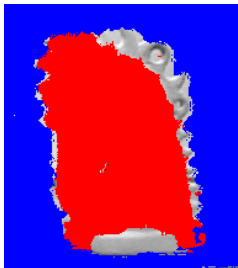
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$$+ \gamma_D \left( \sum_{(p,q \in N)} \left( 1 - \frac{(d_p - d_q)^2}{600 + (d_p - d_q)^2} \right) [x_p \neq x_q] \right).$$

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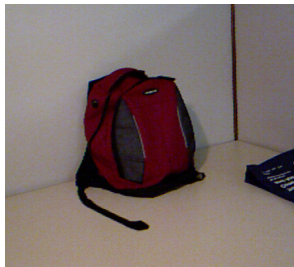
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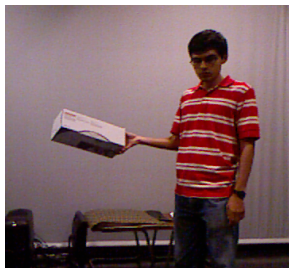


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- First Approach propose a simple and effective method for object extraction in the case of object-background non adjacency. It must improve in depth noise manipulation.
- Second Approach looks for a robust segmentation including data depth in the probabilistic model of GrabCut, but we didn't get an accurate representation of depth distribution from Gaussian Mixture Model
- Third Approach improves the seeding using planar subtraction and gets good results in general cases. It still requires calibration.

## Further Research: Video Object Extraction

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- 5 Return to step 3.

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**Why not include depth data and optical flow data in the energy function?...**

*Thanks to Djalma, Francisco, Leandro, Lucas, and professor Luiz, for suggestions and support!.*

-  Y. Boykov and M-P. Jolly. *Interactive graph cuts for optimal boundary and region segmentation of objects in N-D images*. Proc. IEEE Int. Conf. on Computer Vision, 2001.
-  C. Rother, V. Kolmogorov, and A. Blake. *Grabcut interactive foreground extraction using iterative graph cuts*. Proc. ACM Siggraph, 2004.
-  Y. Boykov and V. Kolmogorov. *An experimental comparison of min-cut/max-flow algorithms for energy minimization in vision*. IEEE Trans. on Pattern Analysis and Machine Intelligence, volume 26, pages 1124-1137, 2004.
-  C. Rother, V. Kolmogorov, Y. Boykov, and A. Blake. *Interactive Foreground Extraction using graph cut*. Microsoft Technical Report:MSR-TR-2011-46.