We study the Riemann random problem for a non-linear system of hyperbolic equations, i.e., we have an equation:

\[ u_t(x, t) + g(u(x, t))_x = 0, \quad t > 0, \quad x \in \mathbb{R}, \]  

(0.1)

where \( u = u(x, t) : \mathbb{R} \times \mathbb{R}^+ \rightarrow \mathbb{Q} \subset \mathbb{R}^n \) are the real state variables and \( g = g(u) = (g_1(u), g_2(u), \ldots, g_n(u))^T : \mathbb{Q} \rightarrow \mathbb{R}^n \) is a continuous field function with first and second derivatives also continuous in \( \mathbb{Q} \), i.e., \( g \in C^2(\mathbb{Q}, \mathbb{R}^n) \).

We assume that the system is provided with random Riemann data of form:

\[ u(x, 0) = \begin{cases} 
  u_L, & \text{if } x < 0, \\
  u_R, & \text{if } x > 0, 
\end{cases} \]  

(0.2)

where \( u_L \) and \( u_R \) are random variables in \( \mathbb{Q} \).

The flux functions are assumed to be known and the randomness appears only on the Riemann data. By assuming that the system is strictly hyperbolic and each field is genuinely non-linear, we find a way to represent the random Riemann solution by using the Riemann solution for the deterministic case. For each \((x^*, t^*)\) fixed and using the self-similarity of the Riemann solution, we obtain a family of sets partitioning the space \((u_L, u_R)\), which is the space of possible random Riemann data, then we extend these regions for all \((x, t) \in \mathbb{R} \times \mathbb{R}^+\). Moreover, by assuming that we know the distribution of the Riemann data on \((u_L, u_R)\) and that the joint probability density function \( f_{LR} \) is known, we obtain an integral representation for the mean value, the variance and higher order statistical moments in the \((u_L, u_R)\)-space.

We give an application of the methodology and theory developed in this paper for a 2 \times 2 system of equations modeling the isentropic flux of gases into a horizontal tube.

We also discuss the Riemann problem for the scalar equation with an inflection point and we show that the numerical results are better than the numerical results find from the Monte Carlo method.

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