Random Lifts of Graphs

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Resumo/Abstract:
Let G, H be finite graphs. A map f from $V(H)$ to $V(G)$ is called a covering map if for every vertex $x \in H$, the mapping f maps the neighbors of x one-to-one onto the neighbors of $f(x)$. It is an easy observation that if G is connected, a covering map to G has a degree n, i.e. f is an $n:1$ mapping both on vertices and on edges. It follows that the vertex set of H is $V(G)x[n]$. The edge set of H has the following form: For every edge $e = xy \in G$ there is a permutation $\pi = \pi_e$ such that for every index j, the vertex (x, j) is adjacent to $(y, pi(j))$. If we select the permutations π_e at random, we say that H is a random n-lift of G. This is an interesting class of random graphs that exhibit random behavior but still reflect the properties of the base graph G. This talk has several parts:

1 For certain graph properties P we ask how likely H is to have property P and how the answer depends on parameters of the base graph G.

2 I will give a brief introduction to eigenvalues of graphs and expander graphs. I will explain how to use random lifts to construct graphs with near-optimal spectral gaps.

3 Finally I will discuss the connection between lifts and the notion of word maps in group theory.