

27° COLOQUIO BRASILEIRO DE MATEMATICA
IMPA, 27 a 31 de julho de 2009

PROPOSTAS DE CURSOS

1. **Titulo de curso:** Algebraic stacks and moduli of vector bundles
2. **Nivel:** Curso Avancado (advanced course for PhD students and researchers)
3. **Autor:** Frank Neumann (Department of Mathematics, University of Leicester, United Kingdom)

4. **Descricao do curso:** The course aims to give a basic introduction to algebraic stacks with a view towards applications to moduli problems. After introducing the general theory of algebraic stacks and discussing important examples like classifying stacks, quotient stacks and moduli stacks of curves, we will concentrate on the particular example of moduli stacks of vector bundles on algebraic varieties. A main goal will be to describe how to calculate the cohomology ring of the moduli stack of vector bundles on algebraic curves. Using the language of algebraic stacks we will indicate how classical calculations of Atiyah-Bott in characteristic zero and Harder-Narasimhan in characteristic p can be unified. If time permits we will also outline how to obtain analogs of the celebrated Weil conjectures for the moduli stack of vector bundles on algebraic curves.

The course will consist of 5 lectures of one hour each distributed in English. The following gives an outline of the distribution of lectures and themes for the course:

Lecture 1: Moduli problems and algebraic stacks I

Themes: Moduli problems, moduli spaces and moduli stacks, sheaves of categories and groupoids, categories fibred in groupoids, Grothendieck topologies, stacks, algebraic stacks, algebraic spaces

Lecture 2: Moduli problems and algebraic stacks II

Themes: Geometry of algebraic stacks, examples: classifying stacks, quotient stacks, moduli stacks of algebraic curves, moduli stacks of vector bundles, moduli of stable and semistable bundles

Lecture 3: Cohomology of algebraic stacks

Themes: Smooth and étale topologies, sheaves for algebraic stacks, sheaf cohomology, Čech cohomology, de Rham cohomology, l -adic cohomology

Lecture 4: Moduli stacks of vector bundles I

Themes: Cohomology of moduli stacks of vector bundles on algebraic curves, Frobenius morphisms

Lecture 5: Moduli stacks of vector bundles II

Themes: Lefschetz trace formula for algebraic stacks, classical Weil conjectures for algebraic varieties, Weil conjectures for the moduli stack of vector bundles on algebraic curves

5. **Pre-requisitos:** Familiarity with basic algebraic geometry: algebraic varieties, algebraic curves, vector bundles; basic notions of schemes, sheaves and cohomology

(e.g. Hartshorne: Chapter 1 and the first half of chapter 2); a few notions from category theory: category, functor, and natural transformation.”.

6. Bibliografia: I plan to write detailed lecture notes for the course. The main references are the following:

1. Artin, M.: Versal deformations and algebraic stacks, *Invent. Math.* 27 (1974), 165-189.
2. Atiyah, M. and Bott, R.: The Yang-Mills equations over Riemann surfaces, *Phil. Trans. Royal Soc. London A* 308 (1982), 523-615.
3. Behrend, K.: The Lefschetz trace formula for algebraic stacks, *Invent. Math.* 112 (1993), 127-149.
4. Deligne, P. and Mumford, D.: The irreducibility of the space of curves of given genus, *Pub. IHES* 36 (1969), 75-109.
5. Esteves, E.: Construção de espaços de moduli, 21º Colóquio Brasileiro de Matemática. IMPA, Rio de Janeiro, 1997.
6. Faltings, G.: Lectures on Vector Bundles on Curves, Notes by Michael Stoll, Universität Bonn 1995
7. Fantechi, B., Göttsche, L., Illusie, L., Kleiman, S., Nitsure, N. and Vistoli, A.: Fundamental algebraic geometry: Grothendieck’s FGA explained, *Math. surveys and Mon.* vol. 123 AMS 2005.
8. Gómez, T.: Algebraic Stacks: *Proc. of the Indian Acad. Sci. (Math. Sci.)* 111, No. 1, February 2001, 1-31.
9. Harder, G. and Narasimhan, M. S.: On the cohomology groups of moduli spaces of vector bundles on curves, *Math. Annalen* 212 (1973), 215-248.
10. Laumon, G. and Moret-Bailly, L.: *Champs algébriques*, *Erg. der Math. und Grenzg.*, 3. Folge, vol. 39, Springer 2000.
11. Neumann, F. and Stuhler, U.: Moduli stacks of vector bundles and Frobenius morphisms. *Algebra and Number Theory, Proc. of the International Conference, HIN/AMS Delhi, 2005*, 126-146.
12. Popp, H.: *Moduli Theory and Classification Theory of Algebraic Varieties*, *Lecture Notes in Math.* vol. 620, Springer 1977.
13. Vistoli, A.: Intersection theory on algebraic stacks and their moduli spaces, *Invent. Math.* 97 (1989), 613-670.

and for background material:

1. Eisenbud, D. and Harris, J.: *The Geometry of Schemes*, *Grad. Texts Maths.* 197, Springer 1999.
2. Griffiths, P. and Harris, J.: *Principles of Algebraic Geometry*, Wiley Intersc. 1978.
3. Hartshorne, R.: *Algebraic Geometry*, *Grad. Texts Maths.* 52, Springer 1977.
4. MacLane, S.: *Categories for the working mathematician*, *Grad. Texts Maths.* 5, Springer 1971.