On certain Neumann problems for Quasilinear Parabolic Systems Modeling Multiphase Flows

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We study the well-posedness of certain Neumann problems for $N \times N$ quasi-linear parabolic systems

$$u_{it} + \text{div} \left( f_i(u) v \right) = \sum_{j=1}^{N} \text{div} \left( B_{ij}(u) \nabla u_j \right), \quad i = 1, \ldots, N,$$

which naturally appear as models in applications such as polydisperse suspensions and multiphase flows in porous media, among others.

We obtain existence and uniqueness of smooth solutions assuming values on a physical invariant domain with no smallness assumption. The boundary value problems considered

$$\left( f_i(u) v(x) - 2 \sum_{j=1}^{2} B_{ij}(u) \nabla u_j \right) \cdot \vec{n}(x) = k_0 u v(x) \cdot \vec{n}(x), \quad i = 1, \ldots, N,$$

include the zero-flux Neumann problem for a model of polydisperse suspensions. The strategy is the use of the Leray-Schauder fixed point theorem, based on a priori estimates implying regularity in Hölder Spaces. As a first step towards the still open general multidimensional case, we consider the case of domains with radial symmetry.

References


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