On towers over finite fields and Ihara's quantity

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Abstract:

Investigating the number of points on an algebraic curve over a finite field is a classical subject in Number Theory and Algebraic Geometry. The origins go back to Fermat, Euler and Gauss, among many others. The basic result is A. Weil’s theorem which is equivalent to the validity of Riemann’s Hypothesis in this context. New impulses came from Goppa’s construction of good codes from curves with many rational points, and also from applications to cryptography. One of the main open problems in this area of research is the determination of Ihara’s quantity $A(\ell)$ for non-square finite fields; i.e., for cardinalities $\ell = p^n$ with $p$ prime and $n$ odd. This quantity controls the asymptotic behaviour of the number of $\mathbb{F}_{\ell}$-rational points (places) on algebraic curves (function fields) as the genus increases towards infinity. This is the topic of the talk.