Vaccination Strategy in SIR Epidemic Model Discrete

This paper deals with the vaccination dynamics of a discrete SIR epidemiological data. A significant aspect of the work is the discrete point of view for studying the SIR epidemiological model that approves the continues models and is more practical to apply the real data. This work is divided in two parts.

In the first part, we developed a mathematical SIR model based on discrete biological data. The model includes the age structure of the human population through the dynamics of the Leslie matrix.

Let $T_n$ be the population size at the day $n$. We denote by $S_n$, $I_n$ and $R_n$ the number of susceptibles, infectives and immunes in the human population at day $n$. Therefore;

$$T_n = S_n + I_n + R_n,$$

The Parameters used in the model are,

- $\mu$: the birth rate and mortality,
- $\lambda$: transmission rate (proportional to the contact between individuals),
- $\beta$: the recovery rate.

The human population will be divided in $\omega$ age classes, numbered from 1 to $\omega$, in such way that the time interval between the classes will be constant and equal to the time interval between the instante $n$ and $n+1$. Since in our model this time interval is of one day, our classes will have only one day. We introduce the following notation:

- $S_n^i$: Number of susceptibles humans in class $i$ at the time instant $n$.
- $I_n^i$: Number of infectives humans in class $i$ at the time instant $n$.
- $R_n^i$: Number of recovered humans in class $i$ at the time instant $n$.
- $T_n^i = S_n^i + I_n^i + R_n^i$: Number of humans in class $i$ at the time instant $n$.

It follows that,

$$S_n = \sum_{i=1}^{\omega} S_n^i, \quad I_n = \sum_{i=1}^{\omega} I_n^i, \quad R_n = \sum_{i=1}^{\omega} R_n^i \quad \text{and} \quad T_n = \sum_{i=1}^{\omega} T_n^i.$$ 

We also introduce the distribution vectors,

$$\vec{S}_n = (S_n^1, S_n^2, ..., S_n^\omega), \quad \vec{I}_n = (I_n^1, I_n^2, ..., I_n^\omega), \quad \vec{R}_n = (R_n^1, R_n^2, ..., R_n^\omega) \quad \text{and} \quad \vec{T}_n = (T_n^1, T_n^2, ..., T_n^\omega).$$

The vital dynamics is introduced through the following coefficients;

- $F^i$: fecundity rate for age class $i$,
- $\mu^i$: mortality rate for age class $i$,
- $P^i = 1 - \mu^i$: survival rate for age class $i$.

The equations are;
\[
\begin{align*}
S_{n+1}^1 &= F_1^1 T_n^1 + \ldots + F_\omega T_n^\omega \\
I_{n+1}^1 &= 0 \\
R_{n+1}^1 &= 0,
\end{align*}
\]  
(0.1)

\[
\begin{align*}
S_{n+1}^i &= P_i^i S_n^i - \frac{\lambda_n^i}{I_n^i} P_i^i S_n^i \\
I_{n+1}^i &= P_i^i I_n^i + \frac{\lambda_n^i}{I_n^i} P_i^i S_n^i - \beta P_i^i I_n^i \\
R_{n+1}^i &= P_i^i R_n^i + \beta I_n^i P_i^i \quad \text{Para} \ 1 < i \leq \omega.
\end{align*}
\]  
(0.2)

Is a structured discrete model with infection.

The above system contains two important embedded models. The first is the Leslie demographic model and is obtained when considering the dynamic for the total population,

\[T_n^i = S_n^i + I_n^i + R_n^i\]

in each class. The second is the discrete SIR model, obtained when we consider the dynamics for \(S_n, I_n\) and \(R_n\) the total number of susceptibles, infectives and removed. Analytical properties of the second embedded model will play a major role in this work.

In the second part two vaccination strategics compared; Constant vaccination and pulse vaccination. We show that under a planned pulse vaccination regime the system converges to a stable solution, the number of infectious individuals being equals to zero. We also show that the pulse vaccination will lead to the elimination of epidemics if certain conditions regarding the magnitude of the rate of vaccination and the duration of the pulses are observed.

Our theoretical results are confirmed by numerical simulations. The introduction of seasonal variation in the basic SIR model leads to periodic and chaotic dynamics of the epidemic. It is shown that under the seasonal variation, despite the complex dynamics of the system, vaccination leads to the eradication of the epidemics. We derived the conditions for the eradication of the epidemic under various constraints and studied the effectiveness and cost of pulse, constant and mixed vaccination policies are compared.

**Constant Vaccination**

\[
\begin{align*}
S_{n+1} &= S_n + \mu T_n - \lambda S_n f(I_n) - \mu S_n - P\mu S_n \\
I_{n+1} &= I_n + \lambda S_n f(I_n) - \mu I_n - \beta I_n \\
R_{n+1} &= R_n + \beta I_n - \mu R_n + P\mu S_n
\end{align*}
\]  
(0.3)