On smooth lattice polytopes with high codegree

Toric geometry provides a bridge between the theory of polytopes and algebraic geometry: there is a one to one correspondence that associates to each lattice polytope $P$ a polarized toric variety $(X, L)$. For instance, there is a combinatorial invariant of the polytope $P$, called codegree, that can be interpreted as a geometric invariant of the pair $(X, L)$, that plays an important role in Adjunction Theory and Mori Theory. Exploring this idea, Dickenstein, Di Rocco and Piene, and later Dickenstein and Nill classified smooth lattice polytopes with codegree $\geq \frac{n+3}{2}$. They belong to a special class of polytopes, called Cayley Polytopes.

In this work we improve the above mentioned result, providing a classification of smooth lattice polytopes with codegree $\geq \frac{n+1}{2}$, under some additional hypothesis.