Explicit bases of Riemann-Roch spaces on an optimal tower of function fields

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For applications in algebraic-geometric codes, an explicit description of bases of Riemann-Roch spaces of divisors on function fields over finite fields is needed. We describe an algorithm to compute such bases and Weierstrass semigroups for some one point divisors over an optimal tower of function fields.

More precisely, we consider the following optimal tower $\left(T_j\right)_{j \geq 0}$ over the finite field $\mathbb{F}_{p^2}$ in odd characteristic. The tower is defined recursively by $T_0 = \mathbb{F}_{p^2}(x_0)$, and $T_{j+1} = T_j(x_{j+1})$, for $j \geq 0$, where the function $x_{j+1}$ satisfies the relation:

$$x_{j+1}^2 = \frac{x_j^2 + 1}{2x_j}.$$

Let $P_{\infty}^j$ be the unique pole in $T_j$ of the function $x_0$. For each $s \in \mathbb{N}$ the relevant Riemann-Roch space is:

$$L(sP_{\infty}^j) = \{z \in T_j \mid \text{the divisor of } z \text{ satisfies } (z) \geq -sP_{\infty}^j\}.$$  

The main result to be presented here is an algorithm to compute bases of the spaces $L(sP_{\infty}^j)$, and the Weierstrass semigroups $H(P_{\infty}^j)$, for any $j$ and $s$. (Joint work with: Gilvan Oliveira - UFES - and Luciane Quoos - UFRJ.)