

A Problem in Probabilistic Combinatorial Number Theory

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Abstract: A celebrated theorem of E. Szemerédi (1975) states that sets of integers with positive upper density contain arbitrarily long arithmetic progressions. This statement for progressions of three elements was established by K.F. Roth (1953).

We may thus say that arithmetic progressions are everywhere within \mathbb{Z} : any non-negligible subset of \mathbb{Z} cannot avoid them. Are there sparser sets of integers with the same property? In fact, do sparse random sets of integers typically satisfy this property? Together with Luczak and Rödl (1996), we showed that this is the case for progressions of length three, and hence a probabilistic generalization of Roth's theorem holds. A challenging problem is to establish the corresponding result for Szemerédi's theorem.