Ergodic Optimization: Maximizing Probabilities, Subactions and L.D.P for Gibbs Probabilities

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Let (X, d) be a compact metric space. If $T : X \to X$ is a continuous function, consider $\mathcal{M}_T$ the set of the T-invariant Borel probability measures.

Consider a fixed continuous Holder function $A : X \to \mathbb{R}$. We denote $\beta_A = \max_{\mu \in \mathcal{M}_T} \int A \, d\mu$. In ergodic optimization on compact spaces, the characterization of the invariant probability measures whose integral of A reaches the maximum value β_A is one of the main goals. We call any of these probabilities an A-maximizing probability and denoted it by μ_∞.

We consider the above problem for expanding transformations T of degree d on the circle, or for the shift on the Bernouilli space $\{1, 2, .., d\}^{\mathbb{N}}$.

Given a potential A, an application $u \in C^0(X)$ is a sub-action for A, if $A + u - u \circ T \leq \beta_A$.

The subactions u play the role in discrete time (symbolic dynamics) of the sub-solutions of the Hamilton-Jacobi equations in Classical Mechanics.

One can consider Gibbs states μ_β associated to the potential $\beta\, A$, where $\beta \in \mathbb{R}$. We describe a Large Deviation Principle for $\mu_\beta \to \mu_\infty$, when $\beta \to +\infty$, in the case the maximizing probability μ_∞ is unique. Subactions play an important role in this result.

Recent developments include duality, the W-kernel, holonomic probabilities, results for Markov chains on the interval, Iterated Function Systems and transport in Ergodic Theory.