On curves and towers over finite fields

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Maximal curves are the ones attaining the famous Hasse-Weil upper bound for the number of rational points on curves over finite fields (Riemann Hypothesis in this context). Ihara has shown that the genus of a maximal curve is upper bounded by half of the cardinality of the finite field, and it is well-known that the Hermitian curve is a maximal curve with the biggest genus possible. Serre has pointed out that subcovers of maximal curves are again maximal, and in particular we get that subcovers of the Hermitian curve are maximal curves. The first example of a maximal curve proven to be not a subcover of the Hermitian curve was found recently by Giulietti and Korchmaros. When we fix the finite field and we consider infinite sequences of curves (the so-called, towers of curves) with increasing genus, then one has that the asymptotic behaviour of the ratios $(\text{number of rational points}) / (\text{genus})$ is upper bounded by the Drinfeld-Vladut bound. We survey on our contributions to maximal curves and to towers of curves over finite fields.