Specific aspects of inverse option pricing: 
nature of ill-posedness and decoupling

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Abstract:

Correct pricing of options and other financial derivatives is of great importance to financial markets and one of the key subjects of mathematical finance. Usually, parameters specifying the underlying stochastic model are not directly observable, but have to be determined indirectly from observable quantities. The identification of local volatility surfaces from market data of European Vanilla options is one very important example of this type. As many other parameter identification problems, the reconstruction of local volatility surfaces is ill-posed, and reasonable results can only be achieved via regularization methods. Moreover, due to sparsity of data, the local volatility is not uniquely determined, but depends strongly on the kind of regularization norm used and a good a-priori guess for the parameter.

Inverse problems in option pricing are frequently regarded as simple and resolved if a formula of Black-Scholes-type defines the forward operator. However, precisely because the structure of such problems is straightforward, they may serve as benchmark problems for studying the nature of ill-posedness and the impact of data smoothness and no arbitrage on solution properties. In the first part of this talk, we analyse the inverse problem of calibrating a purely time-dependent volatility function from a term-structure of option prices by solving an ill-posed nonlinear operator equation in spaces of continuous and power-integrable functions over a finite interval. The forward operator of the inverse problem under consideration is decomposed into an inner linear convolution operator and an outer nonlinear Nemytskii operator given by a Black-Scholes function. The inversion of the outer operator leads to an ill-posedness effect localized at small times, whereas the inner differentiation problem is ill-posed in a global manner. Several aspects of regularization and their properties are discussed. In particular, a detailed analysis of local ill-posedness and Tikhonov regularization of the nonlinear inverse problem including convergence rates is given in a Hilbert space setting. In this context, there are also made some remarks on maximum entropy regularization and its behaviour.

The second part of the talk is concerned with the more general assumption of a multiplicative structure for the local volatility, which is motivated by some specific data situation. Then the inverse problem can be decomposed into two separate subproblems. This removes part of the non-uniqueness and allows to establish convergence and convergence rates under weak assumptions. Additionally, a numerical solution of the two subproblems
is much cheaper than that of the overall identification problem. The theoretical results are illustrated by numerical tests.

Parts of the talk present joint work with Romy Krämer, Torsten Hein (Chemnitz), and Herbert Egger (Linz, Aachen). The research of the author is supported by Deutsche Forschungsgemeinschaft (DFG) under Grant HO1454/7-1.

References