The vertex separation problem: a polyhedral investigation and computations

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The vertex separator (VS) problem in a graph $G = (V, E)$ asks for a partition of $V$ into nonempty subsets $A, B, C$ such that there is no edge between $A$ and $B$, and $|C|$ is minimized subject to a bound on $\max\{|A|, |B|\}$.

We give a mixed integer programming formulation of the problem and investigate the vertex separator polytope (VSP), the convex hull of incidence vectors of vertex separators. Necessary and sufficient conditions are given for the VSP to be full dimensional. Central to our investigation is the relationship between separators and dominators. Several classes of valid inequalities are investigated, along with the conditions under which they are facet defining for the VSP.

Based on the study of the facial structure of the associated polytope, we present separation algorithms for a variety of valid inequalities. Together with a primal heuristics, these algorithms form the main ingredient of a branch-and-cut algorithm that we have implemented for the VSP.

Results obtained by the algorithm for a set of instances related to an application in Linear Algebra and for a well-known graph dataset are discussed.

This is joint work with Egon Balas.