Description of the mini-course "Koszul Duality and Deformation Theory"

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Duration: This course consists of four lectures, to be delivered from Jan. 26 until Jan. 31.

Abstract:

In the sixties, new notions appeared in algebraic topology in order to characterize loop spaces and iterated loop spaces. First Jim Stasheff [8] defined A_∞-structures based on the associahedron (also called the Stasheff polytope) in order to characterize loop spaces. Namely an A_∞-structure is an associative structure up to higher homotopies. It also had many applications in deformation theory of associative algebras. Jim Stasheff went on with the notion of deforming structure as deforming Lie algebras [7], the so-called L_∞-algebras [4]. More recently, in his proof of the Formality Theorem, M. Kontsevich made a great use of L_∞-morphisms [3].

The purpose of this mini-course is to explain how the Koszul duality for operads provides a nice setting to deal with deformations of algebras and algebras up to homotopy in a general context. Before the operadic treatment, a review of the Bar Construction and Koszul Duality for algebras will be given, making this course attractive to non-experts on the subject.

There will be 4 lectures, the first two concentrating on the A_∞-case. The A_∞-case has the advantage of serving as a toy model for operad theory. In the last two lectures it will be shown how to adapt the theorems concerning A_∞-algebras to any algebra over a Koszul operad.

Program:

1- **A_∞-spaces and A_∞-algebras**:
This lecture is an introduction to the work of Stasheff [8]. We will define A_∞-spaces and A_∞-algebras and explain the recognition principle for loop spaces. The main theorems concerning deformations of associative structures will be presented.

2- **Bar construction for algebras and Koszul duality**:
This lecture will be mainly concerned with Koszul duality for associative algebras.

3- **Operads**:
The definitions and examples of topological and algebraic operads [6] will be presented in this lecture. We will explain how Koszul duality for algebras can be written in the language of operads[1, 5].

4- **Algebras up to homotopy and applications to deformation theory**:
We will see some examples of algebras up to homotopy and will show how this concept is used in the study of deformations of algebraic structures. As an application, we will explain how operads and homotopy algebras are used in the proof of the formality theorem [2].

References

1. Victor Ginzburg and Mikhail Kapranov, *Koszul duality for operads*, Duke Math. J. **76** (1994), no. 1, 203–272.
2. Maxim Kontsevich, *Operads and motives in deformation quantization*, Lett. Math. Phys. **48** (1999), no. 1, 35–72, Moshé Flato (1937–1998).
3. ______, *Deformation quantization of Poisson manifolds*, Lett. Math. Phys. **66** (2003), no. 3, 157–216.
4. Tom Lada and Martin Markl, *Strongly homotopy Lie algebras*, Comm. Algebra **23** (1995), no. 6, 2147–2161.
5. Jean-Louis Loday and Bruno Vallette, *Algebraic operads*, Book in preparation, 2010.
6. Martin Markl, Steve Shnider, and Jim Stasheff, *Operads in algebra, topology and physics*, Mathematical Surveys and Monographs, vol. 96, American Mathematical Society, Providence, RI, 2002.
7. Michael Schlessinger and Jim Stasheff, *The Lie algebra structure of tangent cohomology and deformation theory*, J. Pure Appl. Algebra **38** (1985), no. 2-3, 313–322.
8. Jim Stasheff, *Homotopy associativity of H-spaces. I, II*, Trans. Amer. Math. Soc. 108 (1963), 275-292; ibid. **108** (1963), 293–312.